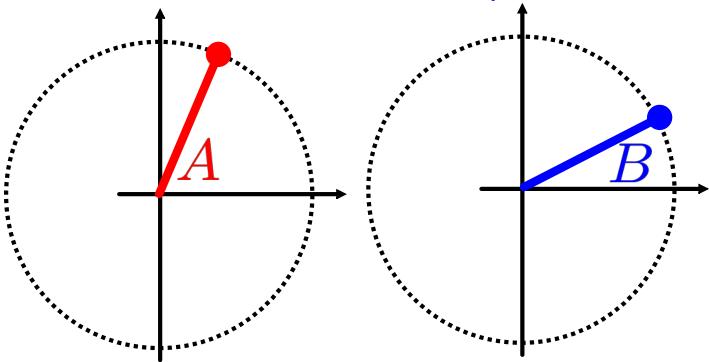


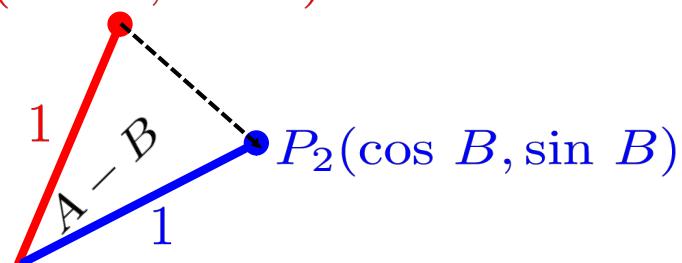
Compound Angle Formulas

Nov. 13/2014

Consider the angles 'A' and 'B' on the unit circle.



$P_1(\cos A, \sin A)$



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$$\text{Cosine Law: } c^2 = a^2 + b^2 - 2ab \cos C$$

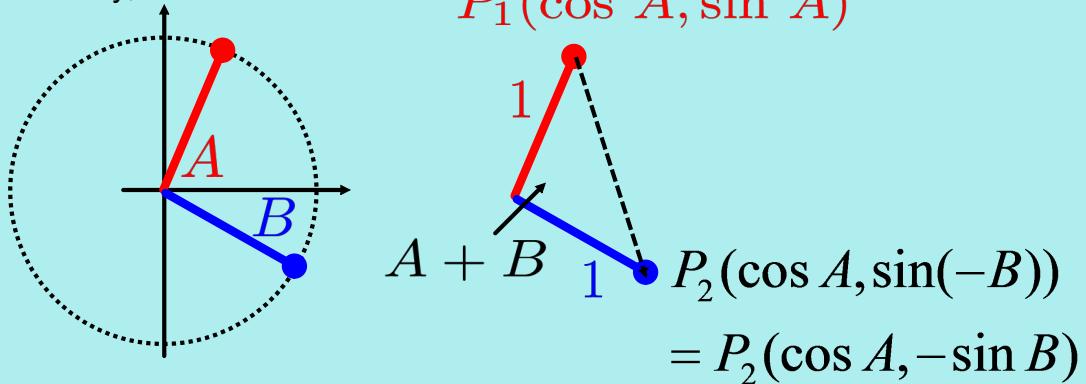
$$\text{Distance Formula: } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use cosine law and distance formula to develop an expression in terms of 'A' and 'B':

$$\begin{aligned}
 d &= \sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2} \\
 d^2 &\quad \text{side across from angle } (A-B) \\
 c^2 &= a^2 + b^2 - 2ab \cos(A-B) \\
 (\cos A - \cos B)^2 + (\sin A - \sin B)^2 &\quad (x-y)^2 \\
 &= 1^2 + 1^2 - 2(1)(1) \cos(A-B) \quad = (x-y)(x-y) \\
 &= 1 - 2 \cos(A-B) \quad = x^2 - 2xy + y^2 \\
 \cancel{\cos^2 A} - 2\cos A \cos B + \cancel{\cos^2 B} + \cancel{\sin^2 A} - 2\sin A \sin B + \cancel{\sin^2 B} &= 1 - 2 \cos(A-B) \\
 &= 2 - 2 \cos(A-B) \\
 2 - 2 \cos A \cos B - 2 \sin A \sin B &= 2 - 2 \cos(A-B) \\
 -2 &\quad -2 \\
 \frac{-2 \cos A \cos B}{-2} - \frac{2 \sin A \sin B}{-2} &= \frac{-2 \cos(A-B)}{-2} \\
 \cos A \cos B + \sin A \sin B &= \cos(A-B)
 \end{aligned}$$

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Similarly,



$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

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Similar compound angle formulas can be obtained for sine using the complimentary angle formula:

$$\sin \theta = \cos \left( \frac{\pi}{2} - \theta \right)$$

Ex.1  $\sin(A + B)$ 

$$\begin{aligned}
 &= \cos \left[ \frac{\pi}{2} - (A + B) \right] \\
 &= \cos \left[ \frac{\pi}{2} - A - B \right] \\
 &= \cos \left[ \left( \frac{\pi}{2} - A \right) - B \right] \\
 &= \cos \left( \frac{\pi}{2} - A \right) \cos B + \sin \left( \frac{\pi}{2} - A \right) \sin B
 \end{aligned}$$

$$= \sin A \cos B + \cos A \sin B$$

$$\therefore \sin(A + B) = \sin A \cos B + \cos A \sin B$$

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For the tangent function, use the quotient identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Ex.2  $\tan(A + B)$

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \times \frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

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Many applications of the compound angle formulas involve angles from the special triangles.

Ex.1 Simplify and then evaluate:

$$\cos \frac{7\pi}{12} \cos \frac{5\pi}{12} + \sin \frac{7\pi}{12} \sin \frac{5\pi}{12}$$

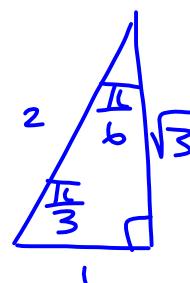
$\cos x \cos y + \sin x \sin y = \cos(x-y)$

$$= \cos \left( \frac{7\pi}{12} - \frac{5\pi}{12} \right)$$

$$= \cos \left( \frac{2\pi}{12} \right)$$

$$= \cos \left( \frac{\pi}{6} \right)$$

$$= \frac{\sqrt{3}}{2}$$



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Ex.2 Determine an exact value for  $\tan\left(\frac{-5\pi}{12}\right)$

notes:

- (1) simplest to convert to RAA and apply CAST
- (2) easier to see sum or difference of special angles by converting to degrees, then back to radians

$$\tan\left(-\frac{5\pi}{12}\right) = -\tan\left(\frac{5\pi}{12}\right)$$

$$\frac{5\pi}{12} \times \frac{180^\circ}{\pi} = 75^\circ$$

$$30^\circ + 45^\circ = 75^\circ$$

$$\frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$$

$$\begin{aligned}\tan\left(-\frac{5\pi}{12}\right) &= -\tan\left(\frac{5\pi}{12}\right) \\ &= -\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\end{aligned}$$

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Assigned Work:

p.400 # 1-4, 5acdf, 6cde, 8, 9, 10, 13

4(a)  $\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$$\begin{aligned}&= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ \\&= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\&= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \\&= \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\&= \frac{\sqrt{2}+\sqrt{6}}{2(2)} \\&= \frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

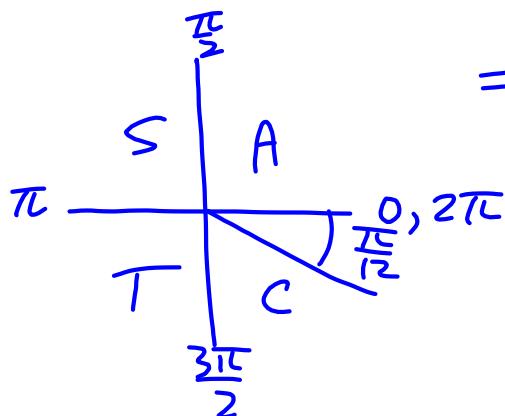
4(c)  $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$$\begin{aligned}&= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}} \\&= \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)(\frac{1}{\sqrt{3}})} \\&= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \\&= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} \\&= \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1} \\&= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\&= \frac{3+\sqrt{3}+\sqrt{3}+1}{3-1} \\&= \frac{4+2\sqrt{3}}{2} \\&= 2+\sqrt{3}\end{aligned}$$

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$$4(f) \tan\left(\frac{23\pi}{12}\right) = -\tan\left(\frac{\pi}{12}\right)$$

$$= -\left[\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right]$$



$$\frac{\pi}{12} = 15^\circ$$

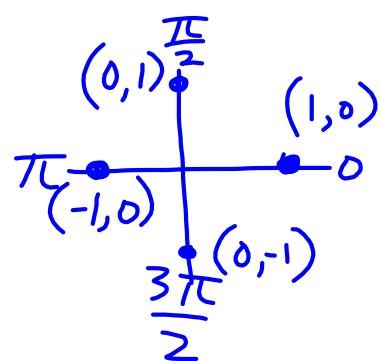
$$45^\circ - 30^\circ \text{ or } 60^\circ - 45^\circ$$

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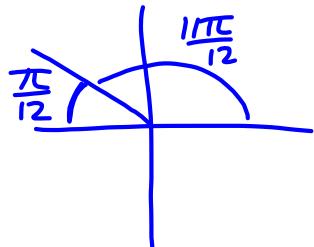
$$6(e) \sin(x - \pi)$$

$$= \sin x \underbrace{\cos \pi}_{-1} - \cos x \underbrace{\sin \pi}_0$$

$$= -\sin x$$



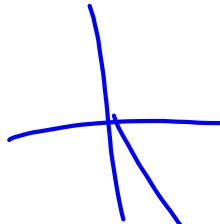
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$$g(c) \cos\left(\frac{11\pi}{12}\right) = -\cos\frac{\pi}{12}$$


$$= -\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= -\left[\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right]$$

$$= \dots$$

$$(f) \tan\left(-\frac{5\pi}{12}\right) = -\tan\left(\frac{5\pi}{12}\right)$$


$$= -\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

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$$9(a) \sin x = \frac{4}{5} \quad \sin y = -\frac{12}{13}$$

↓                    ↓

Q1                    Q4

$$\cos(x+y)$$

$$= \cos x \cos y - \sin x \sin y$$

$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right)$$

$$= \frac{15}{65} + \frac{48}{65}$$

$$= \frac{63}{65}$$

for  $\sin \theta = \frac{4}{5}$

$y = 4$   
 $r = 5$   
 $x = \pm\sqrt{r^2 - y^2}$   
 $= \pm\sqrt{25 - 16}$   
 $= \pm 3$

Q1:  $x = 3$

$\cos \theta = \frac{3}{5}$

for  $\sin \theta = -\frac{12}{13}$

$y = -12$   
 $r = 13$   
 $x = \pm\sqrt{13^2 - (-12)^2}$   
 $= \pm 5$

Q4:  $x = 5$

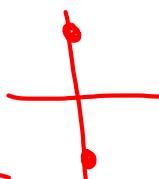
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13.

$$\frac{\sin(f+g) + \sin(f-g)}{\cos(f+g) + \cos(f-g)} \rightarrow \cos(f+g) \neq -\cos(f-g)$$

$$= \frac{\sin f \cos g + \cos f \sin g + \sin f \cos g - \cos f \sin g}{\cos f \cos g - \sin f \sin g + \cos f \cos g + \sin f \sin g}$$

$$= \frac{2 \sin f \cos g}{2 \cos f \cos g} \rightarrow \cos g \neq 0$$

$$= \tan f \quad g \neq \frac{\pi}{2}, \frac{3\pi}{2}$$


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