Double Angle Formulas

Nov. 14/2014

These can be derived from the compound angle formulas for sine, cosine, and tangent.

$$\sin(2\theta) = \sin(\theta + \theta)$$
$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$= 2\sin\theta\cos\theta \checkmark$$

$$\cos(2\theta) = \cos(\theta + \theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= \cos^2\theta - \sin^2\theta \checkmark$$
$$= (1 - \sin^2\theta) - \sin^2\theta$$
$$= 1 - 2\sin^2\theta \checkmark$$
$$= \cos^2\theta - (1 - \cos^2\theta)$$
$$= 2\cos^2\theta - 1 \checkmark$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

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$$\cos(2\theta) = \cos(\theta + \theta)$$

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Ex.1 If $\cos\theta = \frac{-2}{3}$ and $\pi \le \theta < 2\pi$, determine the values of $\sin 2\theta$ and $\cos 2\theta$.

Q3, or ~~Q4~~

cos is positive

$\cos\theta = \frac{-2}{3} \Rightarrow \frac{x}{r} = \frac{-2}{3} \Rightarrow \begin{matrix} x = -2 \\ r = 3 \end{matrix}$

$y = \pm\sqrt{3^2 - (-2)^2}$

$= \pm\sqrt{9-4}$

$= \pm\sqrt{5}$

but θ in Q3

$y = -\sqrt{5}$

$\sin\theta = \frac{-\sqrt{5}}{3}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-\sqrt{5}}{3}\right)\left(\frac{-2}{3}\right)$

$= \frac{4\sqrt{5}}{9}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-2}{3}\right)^2 - \left(\frac{-\sqrt{5}}{3}\right)^2$

$= \frac{4}{9} - \frac{5}{9}$

$= \frac{-1}{9}$

OR

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{-2}{3}\right)^2 - 1$

$= 2\left(\frac{4}{9}\right) - 1$

$= \frac{8}{9} - 1$

$= -\frac{1}{9}$

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Assigned Work:

p.407 # 1-3, 5, 6, 8, 9 (see p.406), 13

2f c

2.(f) $2\tan 60^\circ \cos^2 60^\circ$

$= 2\left(\frac{\sin 60^\circ}{\cancel{\cos 60^\circ}}\right)\cos^{\cancel{2}\,1} 60^\circ$

$= 2\sin 60^\circ \cos 60^\circ$

$= \sin(120^\circ)$

① $\tan\theta = \frac{\sin\theta}{\cos\theta}$

② $\cos^2\theta = 1 - \sin^2\theta$

$2\sin\theta\cos\theta = \sin 2\theta$

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8. $2\tan x - \tan 2x + 2a = 1 - \tan 2x \tan^2 x$

$$2\tan x - \left[\frac{2\tan x}{1-\tan^2 x}\right] + 2a = 1 - \left[\frac{2\tan x}{1-\tan^2 x}\right]\tan^2 x$$

$$\frac{2\tan x(1-\tan^2 x) - 2\tan x}{1-\tan^2 x} + 2a = \frac{1-\tan^2 x - 2\tan^3 x}{1-\tan^2 x}$$

$$2a = \frac{1-\tan^2 x - 2\tan^3 x - [-2\tan^3 x]}{1-\tan^2 x}$$

$$2a = \frac{1-\tan^2 x}{1-\tan^2 x}$$

$2a = 1$, $\tan^2 x \neq 1$

$a = \frac{1}{2}$

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13.(c) $\frac{\pi}{2} \le \theta \le \pi$ Q2 $\sin^2\theta = \frac{8}{9} = \frac{y^2}{r^2}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\cos\left(\frac{\theta}{2}\right)$

$\left(\frac{y}{r}\right)^2 = \frac{8}{9}$

$\frac{y}{r} = \pm\sqrt{\frac{8}{9}}$

$\frac{y}{r} = \pm\frac{2\sqrt{2}}{3}$

$y = \pm 2\sqrt{2}$ Q2 $y = 2\sqrt{2}$

$r = 3$

$x^2 + y^2 = r^2$

$x = \pm\sqrt{9-8}$

$x = \pm 1$ Q2 $x = -1$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$

$\frac{-1}{3} = 2\cos^2\left(\frac{\theta}{2}\right) - 1$

$\frac{2}{3} = 2\cos^2\left(\frac{\theta}{2}\right)$

$\frac{1}{3} = \cos^2\left(\frac{\theta}{2}\right)$

$\cos\left(\frac{\theta}{2}\right) = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$\frac{\frac{\pi}{2}}{2} \le \frac{\theta}{2} \le \frac{\pi}{2}$

$\frac{\pi}{4} \le \frac{\theta}{2} \le \frac{\pi}{2} \Rightarrow \frac{\theta}{2}$ in Q1

$\therefore \cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{3}$

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