Double Angle Formulas

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These can be derived from the compound angle formulas for sine, cosine, and tangent.

$$\sin(2\theta) = \sin(\theta + \theta)$$
$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$= 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos(\theta + \theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= \cos^2\theta - \sin^2\theta \checkmark$$
$$= \cos^2\theta - (1 - \cos^2\theta)$$
$$= 2\cos^2\theta - 1 \checkmark$$
$$= (1 - \sin^2\theta) - \sin^2\theta$$
$$= 1 - 2\sin^2\theta \checkmark$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Nov 13-7:53 PM

$$\cos(2\theta) = \cos(\theta + \theta)$$

Nov 14-8:42 AM

Ex.1 If $\cos\theta = \frac{-2}{3}$ and $\pi \le \theta < 2\pi$, determine the exact values of $\sin 2\theta$ and $\cos 2\theta$.

$\cos\theta = \frac{-2}{3}$

$\Rightarrow \frac{x}{r} = \frac{-2}{3}$

$x = -2$
$r = 3$

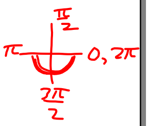
$x^2 + y^2 = r^2$

$y = \pm\sqrt{3^2 - (-2)^2}$

$y = \pm\sqrt{5}$

$\pi \le \theta < 2\pi$

Q3, ~~Q4~~



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$\cos\theta = \frac{-2}{3}$ ~~Q2~~, Q3

$\therefore y = -\sqrt{5}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$

$\boxed{\sin 2\theta = \frac{4\sqrt{5}}{9}}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{-2}{3}\right)^2 - 1$

$= 2\left(\frac{4}{9}\right) - 1$

$= \frac{8}{9} - \frac{9}{9}$

$= \frac{-1}{9}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{-\sqrt{5}}{3}\right)^2$

$= 1 - 2\left(\frac{5}{9}\right)$

$= 1 - \frac{10}{9}$

$= \frac{9}{9} - \frac{10}{9}$

$= \frac{-1}{9}$

Nov 14-8:42 AM

Assigned Work:

p.407 # 1-3, 5, 6, 8, 9 (see p.406), 13

2f

3bd

2(f) $2\tan 60^\circ \cos^2 60^\circ$

$= 2\left(\frac{\sin 60^\circ}{\cancel{\cos 60^\circ}}\right)\cos^{\cancel{2}\,1} 60^\circ$

$= 2\sin 60^\circ \cos 60^\circ$

$= \sin[2(60^\circ)]$

$= \sin 120^\circ$

① $\tan\theta = \frac{\sin\theta}{\cos\theta}$

② $\cos^2\theta = 1 - \sin^2\theta$

Nov 6-9:56 PM

3(b) $\cos 3x$

$= \cos[2(1.5x)]$

$= \cos\left[2\left(\frac{3}{2}x\right)\right]$

$= 2\cos^2\left(\frac{3}{2}x\right) - 1$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - \sin^2\theta$

$2\theta = 3x$

$\theta = \frac{3}{2}x$

3(d) $\cos 6\theta = \cos[2(3\theta)]$

$= \cos^2(3\theta) - \sin^2(3\theta)$

$= 2\cos^2(3\theta) - 1$

$= 1 - 2\sin^2(3\theta)$

Nov 17-12:39 PM

8.

$2\tan x - \underbrace{\tan 2x} + 2a = 1 - \underbrace{\tan 2x}\tan^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$2a = 1 - \tan 2x \tan^2 x - 2\tan x + \tan 2x$

$2a = 1 - \left(\frac{2\tan x}{1-\tan^2 x}\right)\tan^2 x - 2\tan x + \left(\frac{2\tan x}{1-\tan^2 x}\right)$

$2a = \frac{1-\tan^2 x}{1-\tan^2 x} - \frac{2\tan^3 x}{1-\tan^2 x} - \frac{2\tan x(1-\tan^2 x)}{1-\tan^2 x} + \frac{2\tan x}{1-\tan^2 x}$

$2a = \frac{1-\tan^2 x - 2\tan^3 x - 2\tan x + 2\tan^3 x + 2\tan x}{1-\tan^2 x}$

$2a = \frac{1-\tan^2 x}{1-\tan^2 x}$

$2a = 1, \quad \tan^2 x \neq 1$

$a = \frac{1}{2}$

Nov 17-12:44 PM

9. $\sin\left(\frac{\pi}{8}\right) = ?$ $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\frac{\pi}{4} = 2\left(\frac{\pi}{8}\right)$ $\cos\left[2\left(\frac{\pi}{8}\right)\right] = \frac{1}{\sqrt{2}}$

$$1 - 2\sin^2\left(\frac{\pi}{8}\right) = \frac{1}{\sqrt{2}}$$

$$1 - \frac{1}{\sqrt{2}} = 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{\sqrt{2} - 1}{\sqrt{2}} = 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{\sqrt{2} - 1}{2\sqrt{2}} = \sin^2\left(\frac{\pi}{8}\right)$$

discard negative

$$\pm\sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} = \sin\left(\frac{\pi}{8}\right)$$

$\frac{\pi}{8}$ in Q1

$\sin\left(\frac{\pi}{8}\right) > 0$

$$\sin\frac{\pi}{8} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{4}}$$

Nov 17-12:53 PM