

Solving Trigonometric Equations

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

recall: solving linear equations

Solve for x:

$$2x + 3 = 4x - 5$$

recall: related acute angles

recall: CAST Rule

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Solving Linear Trigonometric Equations Nov.19/2014

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

Steps:

1. The equation should involve only one trigonometric ratio .
Use identities, if necessary, to modify the equation.
2. Isolate the trigonometric ratio.
3. Solve for the related acute angle (using positive ratio).
4. Use the actual sign of the ratio to determine the quadrant(s) for your answer [CAST].
5. Determine the angles, within your chosen quadrants, using the related acute angle.
6. Determine any additional coterminal angles, if necessary.

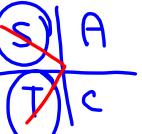
May 4-12:46 PM

Ex.1 Solve $\cos \theta = -\frac{\sqrt{2}}{2}$ where $0 \leq \theta < 2\pi$

① $\cos RAA = \frac{\sqrt{2}}{2}$

$$RAA = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$RAA = \frac{\pi}{4}$$

②  Q2: $\theta = \pi - RAA$
 $= \pi - \frac{\pi}{4}$
 $= \frac{3\pi}{4}$

Q3: $\theta = \pi + RAA$
 $= \pi + \frac{\pi}{4}$
 $= \frac{5\pi}{4}$

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Ex.2 Solve $3\sin 2\theta + 3 = 5$ where $0 \leq \theta < 2\pi$

$$\begin{array}{l} \uparrow \\ \text{Let } a = 2\theta \quad 0 \leq 2\theta < 4\pi \\ \quad \quad \quad 0 \leq a \leq 4\pi \end{array}$$

$$3\sin a + 3 = 5$$

$$3\sin a = 2$$

$$\sin a = \frac{2}{3} \Rightarrow \sin 2\theta = \frac{2}{3}$$

① $RAA = \sin^{-1}\left(\frac{2}{3}\right)$ ② 
 ≈ 0.7297

Since $a = 2\theta$
 $\theta = \frac{a}{2}$

$$\theta_1 = \frac{0.7297}{2}$$

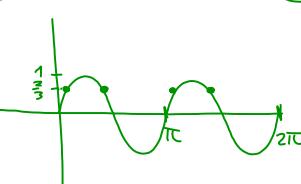
$$\approx 0.3649$$

$$\theta_2 = 1.2060$$

$$\theta_3 = 3.5065$$

$$\theta_4 = 4.3475$$

$$\begin{aligned} \sin 2\theta &= \frac{2}{3} \\ \text{period} &= \frac{2\pi}{k} \\ &= \frac{2\pi}{2} \\ &= \pi \end{aligned}$$



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Assigned Work:

p.426 # 1bdf, 2bdf, 5, 6bdf, 8, 10bef, 11, 18

$$6(f) \tan \theta = \sqrt{3}, \quad 0 \leq \theta \leq 2\pi$$

$$\textcircled{1} \quad \text{RAA} = \tan^{-1}(\sqrt{3})$$

$$= \frac{\pi}{3}$$

$$\textcircled{2} \quad \begin{array}{c} \text{S} \\ \text{A} \end{array} \quad Q1: \theta = \frac{\pi}{3}$$

~~T~~C

$$Q3: \theta = \pi + \frac{\pi}{3} \\ = \frac{4\pi}{3}$$

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$$10(b) \sin 4x = \frac{1}{2} \quad 0 \leq x \leq 2\pi$$

$$\text{let } \theta = 4x$$

$$\sin \theta = \frac{1}{2}$$

$$\textcircled{1} \quad \text{RAA} = \frac{\pi}{6} \quad \textcircled{4} \quad 0 \leq x \leq 2\pi$$

$$\textcircled{2} \quad \begin{array}{c} \text{S} \\ \text{A} \end{array} \quad 0 \leq 4x \leq 8\pi \quad 0 \leq \theta \leq 8\pi$$

$$\textcircled{3} \quad Q1: \theta = \frac{\pi}{6} + 2\pi + 4\pi + 6\pi$$

$$Q2: \theta = \pi - \frac{\pi}{6} \\ = \frac{5\pi}{6} + 2\pi + 4\pi + 6\pi$$

$$Q1: \theta = \frac{\pi}{6}, \frac{13\pi}{6}, \frac{25\pi}{6}, \frac{37\pi}{6}$$

$$4x = \frac{\pi}{6}$$

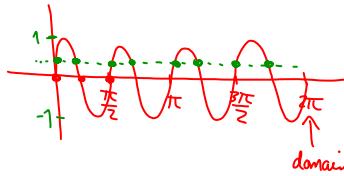
$$x = \frac{\pi}{24}, \frac{13\pi}{24}, \frac{25\pi}{24}, \frac{37\pi}{24}$$

$$Q2: \theta = \frac{5\pi}{6}, \frac{17\pi}{6}, \frac{29\pi}{6}, \frac{41\pi}{6}$$

$$x = \frac{5\pi}{24}, \frac{17\pi}{24}, \frac{29\pi}{24}, \frac{41\pi}{24}$$

$$\sin 4x = \frac{1}{2}$$

$$T = \frac{\pi}{2}$$



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11. $t(d) = -28 \cos\left(\frac{2\pi}{365}d\right) + 10$

Set $t(d) = 32$

$$32 = -28 \cos \theta + 10$$

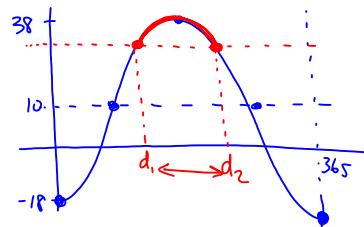
$$-\frac{22}{28} = \cos \theta$$

① RAA = $\cos^{-1}\left(\frac{-22}{28}\right)$
 $\therefore 0.6669$

② $\frac{\sin \theta}{\cos \theta}$ Q2: $\theta = \pi - 0.6669$
 $\therefore 2.4746$

Q3: $\theta = \pi + 0.6669$
 $\therefore 3.8085$

③ $\theta = \frac{2\pi}{365}d$ $d_1 = \underline{\hspace{2cm}}$
 $d = \frac{365\theta}{2\pi}$ $d_2 = \underline{\hspace{2cm}}$



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