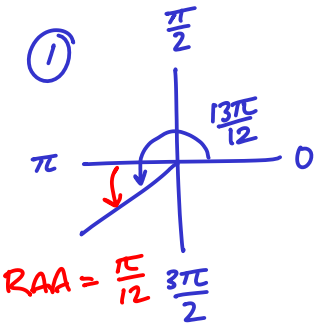
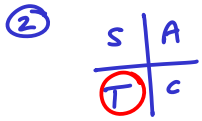
Name: _____ **Section: AM / PM**

MHF4U - Quiz - Equivalent Trigonometric Expressions

1. Determine an exact value for $\sin\left(\frac{13\pi}{12}\right)$ using a compound angle formula. Show your work.

① 

② 

Sine is negative

③ $\sin\left(\frac{13\pi}{12}\right) = -\sin\left(\frac{\pi}{12}\right)$ (from ②, from ①)

④ $\frac{\pi}{12} \times \frac{180^\circ}{\pi} = 15^\circ$

$15^\circ = 45^\circ - 30^\circ$

$\therefore \frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

OR $\left(\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}\right)$

⑤ $= -\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= -\left[\sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}\right]$

$= -\left[\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\right]$

$= -\left[\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right]$

$= \frac{-\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

2. Given $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, determine an exact value for $\cos\frac{\pi}{12}$ using a double angle formula.

① recognize $\frac{\pi}{6} = 2\left(\frac{\pi}{12}\right)$ (2θ, θ)

② want cosine on both sides

$\cos 2\theta = 2\cos^2\theta - 1$

③ $\cos\frac{\pi}{6} = 2\cos^2\left(\frac{\pi}{12}\right) - 1$

$\frac{\sqrt{3}}{2} = 2\cos^2\frac{\pi}{12} - 1$

$\frac{\sqrt{3}}{2} + 1 = 2\cos^2\frac{\pi}{12}$

$\frac{1}{2} \times \left(\frac{\sqrt{3}+2}{2}\right) = \left(2\cos^2\frac{\pi}{12}\right) \times \frac{1}{2}$

$\frac{\sqrt{3}+2}{4} = \cos^2\frac{\pi}{12}$

$\pm\sqrt{\frac{\sqrt{3}+2}{4}} = \cos\frac{\pi}{12}$

④ but $\frac{\pi}{12}$ is an acute angle, Q1, so cosine > 0

$\therefore \cos\frac{\pi}{12} = \sqrt{\frac{\sqrt{3}+2}{4}}$

COMMUNICATION	No Level	0 1 2 3 4	5	6	7	8	9	10
Conventions & Terminology	No level assigned based on content of this page	Unacceptable	Few Major / Many Minor Errors		Few Minor Errors		No Errors	
Expression & Organization			Significant Improvements Required		Few Improvements Required		No Improvements Required	