

Quadratic Trigonometric Equations

Nov. 20/2014

Steps:

- move all the terms to one side of the equal sign so that the equation equals zero
- factor and set each factor to zero, or use the quadratic formula, to solve for the trigonometric ratio
- each factor should involve only one trig ratio
- solve for the RAA for each of the ratios/factors
- apply CAST rule
- determine the angles using RAA
- consider domain and coterminal angles

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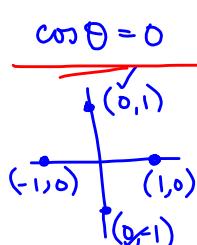
Solving Trigonometric Equations, Part 2

Ex: 1) Solve each equation for $0 \leq \theta < 2\pi$

a) $\cos \theta = 2 \sin \theta \cos \theta$

$$0 = 2 \sin \theta \cos \theta - \cos \theta$$

$$0 = \cos \theta (2 \sin \theta - 1)$$



$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\therefore \theta \text{ is } \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \text{ or } \frac{3\pi}{2}$$

$$2 \sin \theta - 1 = 0$$

$$2 \sin \theta = 1$$

$$\sin \theta = \frac{1}{2}$$

$$\textcircled{1} \quad \text{RAA} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\textcircled{2} \quad \frac{\pi}{6} = \frac{\pi}{6}$$

$$\text{Q1: } \theta = \frac{\pi}{6}$$

$$\text{Q2: } \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

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b) $\sin^2 \theta + 4 \sin \theta = 5$

DO NOT SOLVE

$$\sin^2 \theta + 4 \sin \theta - 5 = 0$$

$$\text{let } x = \sin \theta$$

$$\sin^2 \theta = (\sin \theta)^2$$

$$x^2 + 4x - 5 = 0$$

$$(x+5)(x-1) = 0$$

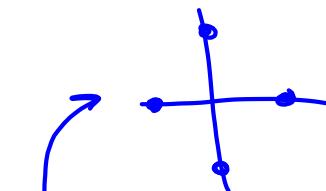
$$x+5=0 \quad \text{or} \quad x-1=0$$

$$x=-5 \quad \text{or} \quad x=1$$

$$\sin \theta = -5 \quad \sin \theta = 1$$

No solution

$$\theta = \frac{\pi}{2}$$



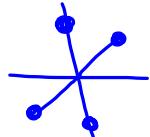
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c) $\sin 2x - 2 \cos^2 x = 0$ (p.428 # 18a)

$$2 \sin x \cos x - 2 \cos^2 x = 0$$

$$2 \cos x (\sin x - \cos x) = 0$$

$$\cos x = 0 \quad \text{OR} \quad \sin x - \cos x = 0$$



$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\tan x = 1$$

$$\textcircled{1} \text{ RAA} = \frac{\pi}{4}$$

$$\textcircled{2} \frac{s(\textcircled{1})}{t(c)}$$

$$\textcircled{3} \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\therefore x \text{ is } \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \text{ or } \frac{3\pi}{2}$$

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Assigned Work:

p.436 # 1, 3, 6ac, 7bd, 8ad, 9ac, 11, 14, 18

- work period Monday
- for Monday, try to complete: 1, 3, 6a, 7b, 8a, 9a
- for Tuesday, try to complete: 6c, 7d, 8d, 9c, 11, 14

$$\begin{aligned}
 1(e) \quad & 24 \sin^2 x - 2 \sin x - 2 \\
 &= 24 a^2 - 2a - 2 \\
 &= 2(12a^2 - a - 1) \quad \begin{array}{l} S = -1 \\ P = -12 \\ I = -4, 3 \end{array} \\
 &= 2(12a^2 - 4a + 3a - 1) \\
 &= 2(4a(3a-1) + 1(3a-1)) \\
 &= 2(3a-1)(4a+1) \\
 &= 2(3 \sin x - 1)(4 \sin x + 1)
 \end{aligned}$$

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8(a) $\sec x \underline{\csc x} - 2 \underline{\csc x} = 0$

$$\csc x (\sec x - 2) = 0$$

↓

$$\begin{aligned}
 \csc x &= 0 \quad \text{or} \quad \sec x - 2 = 0 \\
 \frac{1}{\sin x} &= 0 \\
 \frac{1}{\sin x} &= 0 \quad \frac{1}{\cos x} = 2
 \end{aligned}$$

no solution

$$*\textcircled{1} \quad RAA = \frac{\pi}{3}$$

$$\begin{aligned}
 * \textcircled{2} \quad & Q1: x = \frac{\pi}{3} \checkmark \\
 & Q4: x = 2\pi - \frac{\pi}{3} \star \\
 & \quad = \frac{5\pi}{3} \checkmark
 \end{aligned}$$

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$$b(c) \quad \underbrace{(2\cos x + \sqrt{3})}_{\text{Factor}} \underbrace{\sin x = 0}_{\text{Factor}}$$

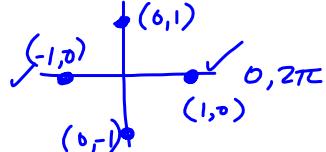
$$2\cos x + \sqrt{3} = 0 \quad \text{or} \quad \sin x = 0$$

$$2\cos x = -\sqrt{3}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\textcircled{1} \text{ RAA} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\text{RAA} = \frac{\pi}{6}$$



$$x = 0 \quad \checkmark$$

$$x = \pi \quad \checkmark$$

$$\textcircled{2} \quad \begin{array}{l} \textcircled{1} \text{ A} \\ \textcircled{2} \text{ C} \end{array} \quad Q2: x = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \quad 0 \leq x \leq 2\pi \quad x = 2\pi \quad \checkmark$$

$$Q3: x = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \quad \checkmark$$

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$$8(d) \quad 2\cot x + \sec^2 x = 0$$

$$\frac{2}{\tan x} + (1 + \tan^2 x) = 0 \quad [\times \tan x]$$

$$2 + \tan x + \tan^3 x = 0$$

$$\tan^3 x + \tan x + 2 = 0$$

$$\text{let } A = \tan x \quad OA^2$$

$$A^3 + A + 2 = 0$$

$$A = -1 \Rightarrow A+1 \text{ is a factor}$$

$$\begin{array}{r} -1 \\ \hline 1 & 0 & 1 & 2 \\ \downarrow & & -1 & 1 \\ 1 & -1 & 2 & 0 \end{array}$$

$$b^2 - 4ac = (-1)^2 - 4(1)(2) = -7$$

$$\rightarrow (A+1)(A^2 - A + 2) = 0$$

no zeros

$$A+1=0$$

$$\tan x + 1 = 0$$

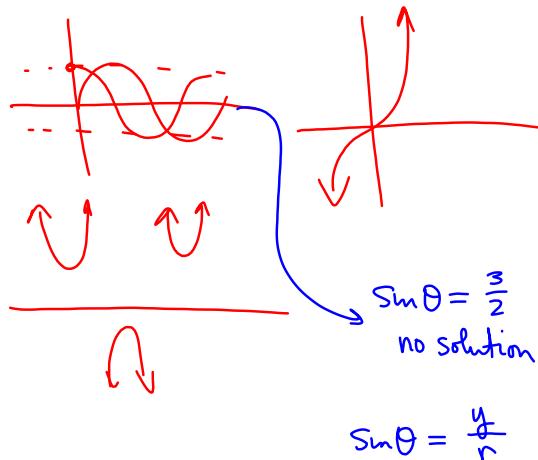
$$\tan x = -1$$

$$\textcircled{1} \text{ RAA}$$

$$\textcircled{2} \text{ CAST}$$

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$$\begin{aligned}
 14. \quad 6\sin^2 x &= 17\cos x + 11 \\
 6\sin^2 x - 17\cos x - 11 &= 0 \\
 6(1-\cos^2 x) - 17\cos x - 11 &= 0 \\
 -6\cos^2 x - 17\cos x - 5 &= 0 \quad [\times (-1)] \\
 6\cos^2 x + 17\cos x + 5 &= 0 \\
 6A^2 + 17A + 5 &= 0 \quad \begin{array}{r} S \ 17 \\ P \ 30 \\ \hline I \ 2, 15 \end{array}
 \end{aligned}$$



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Review

p.440 # 1, 2, 3ac, 4, 5, 6, 7, 8, 9, 10, 11b, 12bcd, 13
c

$$9. \quad \frac{2\sec^2 x - 2\tan^2 x}{\csc x} = \sin 2x \sec x$$

$$\begin{aligned}
 LS &= \frac{2\sec^2 x - 2(\sec^2 x - 1)}{\csc x} \\
 &= \frac{2\sec^2 x - 2\sec^2 x + 2}{\csc x} \\
 &= \frac{2}{\frac{1}{\sin x}} \\
 &= 2\sin x \\
 RS &= \sin 2x \sec x \\
 &= \frac{2\sin x \cos x}{\cos x} \\
 &= 2\sin x, \cos x \neq 0
 \end{aligned}$$

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$$10(c) \quad 3 + 10 \sec x - 1 = -18$$

$$\frac{10 \sec x}{10} = \frac{-20}{10}$$

$$\sec x = -2$$

$$\frac{1}{\cos x} = -2$$

$$\frac{1}{-2} = \frac{-2 \cos x}{-2}$$

$$\cos x = \frac{-1}{2}$$

① RAA

② CAST

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