

Evaluating Logarithms

Dec 2/2014

$$y = \log_a x \text{ is equivalent to } x = a^y$$

For many problems, we can obtain an exact value by switching between these equivalent expressions and looking for a common base.

There are also some general rules we can develop.

Ex.1 Solve

(a)  $y = \log_3 3^2$

$$3^2 = 3^y$$

$$y = 2$$

(b)  $y = \log_4 4^7$

$$4^7 = 4^y$$

$$y = 7$$

Nov 27-7:50 PM

In general:

$$\log_a a^x = x \quad (1)$$

Ex.2 Evaluate:

(a)  $\log_{10} 1$

$$y = \log_{10} 1$$

$$10^y = 1$$

$$10^y = 10^0$$

$$y = 0$$

(b)  $\log_5 1$

$$y = \log_5 1$$

$$5^y = 1$$

$$5^y = 5^0$$

$$y = 0$$

Dec 1-7:19 PM

In general:

$$\log_a 1 = 0 \quad (2)$$

Ex.3 Evaluate:

(a)  $2^{\log_2 x}$

$$2^y = 2^{\log_2 x}$$

$$y = \log_2 x$$

$$x = 2^y$$

$$x = 2^{\log_2 x}$$

(b)  $5^{\log_5 x}$

$f(x)$  and  $f^{-1}(x)$   
undo each other.

$$f(x) = 5^x$$

$$f^{-1}(x) = \log_5 x$$

$$f(f^{-1}(x)) = x$$

$$5^{\log_5 x} = x$$

$$\log_5 5^x = x$$

Dec 1-7:22 PM

In general:

$$a^{\log_a x} = x \quad (3)$$

What if no common base is possible, and these general rules cannot be applied?

Recall: Many calculators only allow for a base of 10 or 'e'.

$$y = \log_{10} x \quad \text{or} \quad y = \ln x$$

For different bases, we can still calculate the value of a logarithm by using an equivalent expression.

$$\log_a x = \frac{\log_{10} x}{\log_{10} a} \quad (4)$$

Note: we will derive this in our lesson on "laws of logarithms"

Dec 1-7:26 PM

Assigned Work:

p.466 # 3 (using rule 1 or 2)

5 (using 1 or 2)

8 (using 4)

6, 9, 12, 17

$\frac{c}{a} = b$

$$50 = 2^x$$

$$x = \log_2 50$$

$$x = \frac{\log_{10} 50}{\log_{10} 2}$$

$$x \doteq$$

Nov 27-8:36 PM

9(a)  $4^{\log_4 \frac{1}{16}} = \frac{1}{16}$

$a^{\log_a x} = x$

$$\begin{aligned} 4^{\log_4(4^{-2})} &= 4^{-2} \\ &= \frac{1}{16} \end{aligned}$$

(e)  $a^{\log_a b} = b$

Dec 3-10:35 AM

$$\begin{aligned}
 12 \quad m(t) &= P\left(\frac{1}{2}\right)^{\frac{t}{h}} \\
 &= P\left(\frac{1}{2}\right)^{\frac{t}{1620}} \\
 (a) \quad m(t) &= S\left(\frac{1}{2}\right)^{\frac{t}{1620}} \\
 m(150) &= S\left(\frac{1}{2}\right)^{\frac{150}{1620}} \\
 (b) \quad m(t) &= 4 \quad \text{solve for } t \\
 4 &= S\left(\frac{1}{2}\right)^{\frac{t}{1620}} \\
 0.8 &= \left(\frac{1}{2}\right)^{\frac{t}{1620}} \\
 \frac{t}{1620} &= \log_{0.5} 0.8 \\
 \frac{t}{1620} &= \frac{\log_{10} 0.8}{\log_{10} 0.5} \\
 t &= 1620 \left( \frac{\log 0.8}{\log 0.5} \right) \\
 &\doteq 522 \\
 \therefore & 522 \text{ years.}
 \end{aligned}$$

Dec 3-10:37 AM

$$\begin{aligned}
 17. \quad y &= y_0 2^{\frac{t}{D}} \\
 &= y_0 2^{\frac{t}{0.32}} \\
 (a) \quad y &= 100 \left( 2^{\frac{t}{0.32}} \right) \checkmark \\
 \frac{y}{100} &= 2^{\frac{t}{0.32}} \quad \leftarrow \text{rewrite for log} \\
 y &= a^x \\
 (d) \quad \frac{t}{0.32} &= \log_2 \left( \frac{y}{100} \right) \\
 (e) \quad \text{set } y &= 450 \\
 \frac{t}{0.32} &= \log_2 \left( \frac{450}{100} \right)
 \end{aligned}$$

Dec 3-10:43 AM