Evaluating Logarithms

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$$\boxed{y = \log_a x \text{ is equivalent to } x = a^y}$$

For many problems, we can obtain an exact value by switching between these equivalent expressions and looking for a common base.

There are also some general rules we can develop.

Ex.1 Solve

(a) $y = log_3 3^2$

$3^y = 3^2$

$y = 2$

(b) $y = log_4 4^7$

$4^y = 4^7$

$y = 7$

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In general: $\boxed{log_a a^x = x}$ (1)

Ex.2 Evaluate:

(a) $log_{10} 1$

$y = \log_{10} 1$

$10^y = 1$

$10^y = 10^0$

$y = 0$

(b) $log_5 1$

$= \log_5 5^0$

$= 0$

$\log_a 1 = 0$

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In general: $\boxed{log_a 1 = 0}$ (2)

Ex.3 Evaluate:

(a) $2^{log_2 x}$ (b) $5^{log_5 x} = x$

let $y = \log_2 x$

$x = 2^y$

$x = 2^{\log_2 x}$

$2^{\log_2 x} = x$

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In general: $\boxed{a^{log_a x} = x}$ (3)

What if no common base is possible, and these general rules cannot be applied?

Recall: Many calculators only allow for a base of 10 or 'e'.

$y = \log_{10} x$ or $y = \ln x$

For different bases, we can still calculate the value of a logarithm by using an equivalent expression.

$$\boxed{\log_a x = \frac{\log_{10} x}{\log_{10} a}}$$ (4)

Note: we will derive this in our lesson on "laws of logarithms"

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Assigned Work:

p.466 # 3 (using rule 1 or 2) d
5 (using 1 or 2) c e
8 (using 4) e
6, 9, 12, 17

3(d) $\log_7 \sqrt{7} = \log_7 7^{\frac{1}{2}}$
$= \frac{1}{2}$

5(c) $\log_3 81 + \log_4 64$
$= \log_3 3^4 + \log_4 4^3$
$= 4 + 3$
$= 7$

6(e) $\log_5 x = \frac{1}{2}$
$x^{\frac{1}{2}} = 5$
$\sqrt{x} = 5$
$x = 25$

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9(e) $a^{\log_a b} = y$
$\log_a y = \log_a b$
$y = b$

12 $m(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$
$m(t) = (5)\left(\frac{1}{2}\right)^{\frac{t}{1620}}$

(a) set $t = 150$
$m(150) = 5\left(\frac{1}{2}\right)^{\frac{150}{1620}}$
$\doteq$ ______

(b) $m(t) = 4$
$4 = 5\left(\frac{1}{2}\right)^{\frac{t}{1620}}$
$0.8 = \left(\frac{1}{2}\right)^{\frac{t}{1620}}$

$y = a^x$
$x = \log_a y$

$\frac{t}{1620} = \log_{0.5} 0.8$
$\frac{t}{1620} = \frac{\log_{10} 0.8}{\log_{10} 0.5}$
$t = 1620\left(\frac{\log 0.8}{\log 0.5}\right)$

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17. a d e

$$y = y_0 \, 2^{\frac{t}{D}}$$

(a) $y = 100\left(2^{\frac{t}{0.32}}\right)$

(d) $\frac{y}{100} = 2^{\frac{t}{0.32}}$ $\quad y = a^x$

$$\frac{t}{0.32} = \log_2\left(\frac{y}{100}\right) \quad \text{ok} \checkmark$$

$$t = 0.32 \log_2\left(\frac{y}{100}\right) \checkmark$$

(e) want $y = 450$

$$t = 0.32 \log_2\left(\frac{450}{100}\right)$$

$$= 0.32 \log_2(4.5)$$

$$= 0.32\left[\frac{\log_{10} 4.5}{\log_{10} 2}\right]$$

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