

Laws of Logarithms

Dec 3/2014

Recall: Exponent Laws

same base

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, a \neq 0$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1, a \neq 0$$

different bases

$$(ab)^x = (a^x)(b^x)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, b \neq 0$$

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Ex. Derive the Product Law for Logarithms
(use the product law for exponents) $(a^x)(a^y) = a^{x+y}$

$$\text{let } m = a^x, n = a^y$$

$$mn = a^{x+y}$$

$$x+y = \log_a(mn)$$

$$y = a^x$$

$$m = a^x$$

$$n = a^y$$

$$x = \log_a m$$

$$y = \log_a n$$

$$\log_a m + \log_a n = \log_a(mn)$$

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Ex. Derive the Quotient Law for Logarithms $\frac{a^x}{a^y} = a^{x-y}$
 (use the quotient law for exponents)

$$\text{let } m = a^x, n = a^y$$

$$x = \log_a m$$

$$y = \log_a n$$

$$\frac{m}{n} = a^{x-y}$$

$$x-y = \log_a \left(\frac{m}{n} \right)$$

$$\log_a m - \log_a n = \log_a \left(\frac{m}{n} \right)$$

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Ex. Derive the Power Law for Logarithms
 (use the power law for exponents)

$$(a^x)^y = a^{xy}$$

$$\text{let } m = a^x, n = y$$

$$x = \log_a m$$

$$m^n = a^{xy}$$



keep base a

$$xy = \log_a (m^n)$$

$$(\log_a m)(n) = \log_a (m^n)$$

$$n \log_a m = \log_a (m^n)$$

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product law: $\log_a xy = \log_a x + \log_a y$

quotient law: $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$

power law: $\log_a x^r = r \log_a x$

Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

(b) $\log_2 48 - \log_2 3$

(c) $\log_5 \sqrt[3]{25}$

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Ex.1 Simplify then evaluate (no log calculations!):

(a) $\log_3 6 + \log_3 4.5$

(b) $\log_2 48 - \log_2 3$

(c) $\log_5 \sqrt[3]{25}$

(a) $\log_3 6 + \log_3 4.5$
 $= \log_3 (6(4.5))$
 $= \log_3 27$
 $= \log_3 3^3$
 $= 3$

(b) $\log_2 48 - \log_2 3$
 $= \log_2 \left(\frac{48}{3} \right)$
 $= \log_2 16$
 $= \log_2 2^4$
 $= 4$

(c) $\log_5 \sqrt[3]{25}$
 $= \log_5 25^{\frac{1}{3}}$
 $= \frac{1}{3} \log_5 25$
 $= \frac{1}{3} \log_5 5^2$
 $= \frac{1}{3} (2)$
 $= \frac{2}{3}$

$\log_a m^r = r \log_a m$

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Ex.2 Use the power law to show $\log_a x = \frac{\log_{10} x}{\log_{10} a}$

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Ex.3 Rewrite as a single log to a common base:

$$\begin{aligned} & \log 12 + \frac{1}{2} \log 7 - \log 2 \\ &= \log 12 + \log 7^{\frac{1}{2}} - \log 2 \\ &= \underbrace{\log 12 + \log \sqrt{7}} - \log 2 \\ &= \log (12\sqrt{7}) - \log 2 \\ &= \log \left(\frac{12\sqrt{7}}{2} \right) \\ &= \log (6\sqrt{7}) \end{aligned}$$

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Assigned Work:

p.475 # 5 (look past obvious answer!),

6, 7, 9ace, 10ac, 11a, 12, 17

c d
d

$$\begin{aligned} 6(c) \quad \log_6(6\sqrt{6}) &= \log_6(6^1 6^{\frac{1}{2}}) \\ &= \log_6(6^{\frac{3}{2}}) \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} (d) \quad \log_2 \sqrt{36} - \log_2 \sqrt{72} \\ &= \log_2 36^{\frac{1}{2}} - \log_2 72^{\frac{1}{2}} \\ &= \frac{1}{2} \log_2 36 - \frac{1}{2} \log_2 72 \\ &= \frac{1}{2} (\log_2 36 - \log_2 72) \\ &= \frac{1}{2} \log_2 \left(\frac{36}{72} \right) \\ &= \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \\ &= \frac{1}{2} \log_2 (2^{-1}) \\ &= \frac{1}{2} (-1) \\ &= -\frac{1}{2} \end{aligned}$$

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$$\begin{aligned} 7(d) \quad \log_b \sqrt{x^5 y z^3} \\ &= \log_b (x^5 y z^3)^{\frac{1}{2}} \\ &= \frac{1}{2} \log_b (x^5 y z^3) \\ &= \frac{1}{2} \left[\log_b x^5 + \log_b y + \log_b z^3 \right] \\ &= \frac{1}{2} \left[5 \log_b x + \log_b y + 3 \log_b z \right] \end{aligned}$$

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$$10(e) \quad \log_3 x = 2 \log_3 10 - \log_3 25$$

$$\log_3 x = \log_3 10^2 - \log_3 25$$

$$\log_3 x = \log_3 \left(\frac{10^2}{25} \right)$$

$$\log_3 x = \log_3 \left(\frac{100}{25} \right)$$

$$\log_3 x = \log_3 4$$

$$\boxed{x = 4}$$

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$$11.(b) \quad \log_5 u - \log_5 v + \log_5 w$$

$$= \log_5 \left(\frac{u}{v} \right) + \log_5 w$$

$$= \log_5 \left(\frac{uw}{v} \right) \quad \left(\frac{u}{v} \right) \left(\frac{w}{1} \right)$$

$$(d) \quad \log_2 x^2 - \log_2 xy + \log_2 y^2$$

$$= \log_2 \left(\frac{x^2}{xy} \right) + \log_2 y^2$$

$$= \log_2 \left(\frac{x^2 y^2}{xy} \right)$$

$$= \log_2 (xy)$$

$$= 2 \log_2 x - (\log_2 x + \log_2 y) + 2 \log_2 y$$

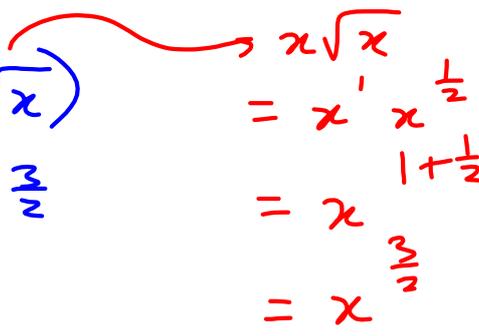
$$= 2 \log_2 x - \log_2 x - \log_2 y + 2 \log_2 y$$

$$= \log_2 x + \log_2 y$$

$$= \log_2 (xy)$$

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$$17. \quad \underline{\log_b x = 0.3}$$

$$\begin{aligned} \log_b(x\sqrt{x}) &= \log_b x^{1+\frac{1}{2}} \\ &= \log_b x^{\frac{3}{2}} \\ &= \frac{3}{2} \log_b x \\ &= \frac{3}{2} (0.3) \\ &= 0.45 \end{aligned}$$


Handwritten red annotations showing the conversion of $x\sqrt{x}$ to $x^1 x^{\frac{1}{2}}$ and then to $x^{1+\frac{1}{2}}$ and finally to $x^{\frac{3}{2}}$.

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