Solving Exponential and Logarithmic Equations Dec 4/2014

The definition and properties of logarithms can be used to solve equations in which either powers or logarithms appear. If the unknown occurs in an exponent then the strategy is to isolate it by taking the logarithm of both sides.

Ex.1 Solve $3^{x+2} = 4$
(a) using definition of logarithms.
(b) by taking the log (base 10) of both sides.

check your solution!

(a) $\log_3 4 = x+2$

$x = a^y$
$y = \log_a x$

$x+2 = \log_3 4$

$x = \log_3 4 - 2$

$x = \frac{\log_{10} 4}{\log_{10} 3} - 2$

$x \doteq -0.7381$

(b) $3^{x+2} = 4$

$x = 2$
$2^x = 2^2$

$\log(3^{x+2}) = \log 4$

$(x+2)\frac{\log 3}{\log 3} = \frac{\log 4}{\log 3}$

$x+2 = \frac{\log 4}{\log 3}$

$x = \frac{\log 4}{\log 3} - 2$

Dec 3-8:13 PM

Ex.2 Solve $\log_2 x - \log_2 3 = \log_2 6$ two ways.

(a) $\log_2 x - \log_2 3 = \log_2 6$

$\log_2 x = \log_2 6 + \log_2 3$

$\log_2 x = \log_2 (6 \cdot 3)$

$\log_2 x = \log_2 (18)$

$\boxed{x = 18}$

(b) $\log_2 x - \log_2 3 = \log_2 6$

$\log_2 \left(\frac{x}{3}\right) = \log_2 6$

$\frac{x}{3} = 6$

$x = 18$

Dec 3-8:19 PM

Ex.3 Solve $6^{3x} = 4^{2x-3}$

$\log_{10}(6^{3x}) = \log_{10}(4^{2x-3})$

$3x \underbrace{\log 6}_{\text{number}} = (2x-3) \underbrace{\log 4}_{\text{number}}$

$3x \log 6 = 2x \log 4 - 3 \log 4$

$3x \log 6 - 2x \log 4 = -3 \log 4$

$x(3 \log 6 - 2 \log 4) = -3 \log 4$

$x = \dfrac{-3 \log 4}{3 \log 6 - 2 \log 4}$

$x \doteq -1.598$

$LS \doteq 6^{3(-1.598)}$

$\doteq 1.86 \times 10^{-4}$

$RS \doteq 4^{2(-1.598)-3}$

$\doteq 1.86 \times 10^{-4}$

Dec 3-8:25 PM

Ex.4 Solve $\log_x 0.04 = -2$

$y = a^x \quad a > 0, \ a \neq 1$

$x = \log_a y$

$x^{-2} = 0.04$

$\frac{1}{x^2} = 0.04$

$x^2 = \frac{1}{0.04}$

$x^2 = 25$

$x = \pm 5, \quad x > 0 \ (\text{base} > 0)$

$x = 5$

Dec 3-8:32 PM

Ex.5 Solve $\log(x+2) + \log(x-1) = 1$

check your solution!

$\log[(x+2)(x-1)] = 1$

$1 = \log_{10} 10$

$\log[(x+2)(x-1)] = \log 10$

$\log A = \log B$

$A = B$

$(x+2)(x-1) = 10$

$x^2 + x - 2 - 10 = 0$

$x^2 + x - 12 = 0$

$(x-3)(x+4) = 0$

$x = 3$ or $x = -4$

$LS = \log(3+2) + \log(3-1)$

$= \log 5 + \log 2$

$= \log 10$

$= 1$

$= RS$ ✓

$LS = \log(-4+2) + \log(-4-1)$

$= \log(-2) + \log(-5)$

$\cancel{= \log[(-2)(-5)]}$

cannot take log of negative

everything after is invalid

discard $x = -4$

$\therefore$ Solution is $x = 3$

$2\log(-2) = \cancel{\log(-2)^2}$

$\cancel{= \log 4}$

Dec 3-8:23 PM

Assigned Work:

p.485 # 2, 8a, 10, 17
p.491 # 4, 5, 7, 12 — for tomorrow

p.485 # 4, 6e, 6c, 7, 11
p.492 # 3, 9a — work period?

$P(t) = P_0(2)^{\frac{t}{D}}$

P.485

8(a) $4^{x+1} + 4^x = 160$

$4^x 4^1 + 4^x = 160$

$4^x(4+1) = 160$

$4^x = 32$

$\log 4^x = \log 32$

$x \log 4 = \log 32$

$x = \dfrac{\log 32}{\log 4}$

$x = 2.5$

$(2^2)^x = 2^5$

$2^{2x} = 2^5$

$2x = 5$

$x = \frac{5}{2}$

Dec 3-8:25 PM

p.485

8(c) $2^{x+2} - 2^x = 96$

$$2^x 2^2 - 2^x = 96$$

$$2^x(2^2-1) = 96$$

$$2^x(3) = 96$$

$$2^x = 32$$

$$2^x = 2^5$$

$$x = 5$$

Dec 5-11:02 AM

p.491

5(e) $3\log x - \log 3 = 2\log 3$

$$\log x^3 = \log 3^2 + \log 3$$

$$\log x^3 = \log 3^3$$

$$x = 3$$

$$\log x^3 = \log 27$$

$$x^3 = 27$$

$$x = \sqrt[3]{27}$$

$$x = 3$$

Dec 5-11:03 AM

1. $4^{\log_4 64} + 10^{\log_{10} 100}$

$= 64 + 100$

$= 164$

2. $\log_5 625 + \log_2 32$

$= \log_5 5^4 + \log_2 2^5$

$= 4 + 5$

$= 9$

3. $4\log 2 + \log 6 - \log 3$

$= \log 2^4 + \log\left(\frac{6}{3}\right)$

$= \log 2^4 + \log 2$

$= \log(2^4 \cdot 2)$

$= \log(2^5)$ ✓

$= 5\log 2$ ✓ ok

$= \log 32$ ✓

Dec 5-10:35 AM

p. 485

4. $m(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$h = 8h$

$P = 300$

(a) $300 \to 200$ $M(t) = 200,\ t = ?$

$200 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\frac{2}{3} = \left(\frac{1}{2}\right)^{t/8}$

$\log\left(\frac{2}{3}\right) = \log\left(\frac{1}{2}\right)^{t/8}$

$\log\left(\frac{2}{3}\right) = \frac{t}{8}\log\left(\frac{1}{2}\right)$

$\frac{\log\left(\frac{2}{3}\right)}{\log\left(\frac{1}{2}\right)} = \frac{t}{8}$

$t = \frac{8\log\left(\frac{2}{3}\right)}{\log\left(\frac{1}{2}\right)}$

$t \doteq 4.68$

$\therefore$ ______

Dec 8-9:13 AM

p.485
6(b)

$$A = P(1+i)^n$$

$$i = \frac{0.12}{12}$$

$n = ?$ # of compounding periods

$A = 5000$
$P = 1000$

$$5000 = 1000\left(1+\frac{0.12}{12}\right)^n$$

$$5 = (1.01)^n$$

$$\log 5 = \log (1.01)^n$$

$$\log 5 = n \log (1.01)$$

$$n = \frac{\log 5}{\log 1.01}$$

$n \doteq 161.7$ (# of months)

$\div 12$ to get answer in years.

Dec 8-9:18 AM

p.485
11(a)

thickness	intensity
1mm	0.95
2mm	$(0.95)(0.95)$ 1mm 1mm
3mm	$(0.95)^3$

(a) $I(T) = (0.95)^T$

intensity thickness

(b) set $I(T) = 0.60$

$$0.60 = (0.95)^T$$

$$\log 0.60 = \log 0.95^T$$

$$T = \frac{\log 0.60}{\log 0.95}$$

Dec 8-9:23 AM

p.492

#9. $L = 10 \log\left(\frac{I}{I_0}\right)$

$I_0 = 10^{-12}$

(a) $L = 50$ dB

$50 = 10 \log\left(\frac{I}{10^{-12}}\right)$

$5 = \log_{10} \frac{I}{10^{-12}}$

$y = a^x$

$x = \log_a y$

$10^5 = 10^{\log \frac{I}{10^{-12}}}$

$100\,000 = \frac{I}{10^{-12}}$

$I = 10^{-7}$

$\frac{I}{10^{-12}} = 10^5$

$I = 10^{-7}$

$\therefore$ ~~~~~~

Dec 8-9:27 AM