Solving Problems with Logarithmic Functions

Dec 8/2014

pH Scale (hydrogen ion concentration):

$$pH = -\log_{10} H^{+}$$

where pH is the scaled measurement (0 to 14)
and H+ is the concentration of hydrogen ions (mol/L)
(see p.494 for pH scale examples)

Ex. Calculate the pH for a hydrogen ion concentration of 0.00025 mol/L. Is it an acid or base?

$pH = -\log(0.00025)$

$pH \doteq 3.6$, acid

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Richter Scale (earthquakes):

$$M = \log_{10} A$$

where M is the magnitude (approximately 0 to 10)
and A is the amplitude on the seismograph

Note: This formula is useful only for comparing the relative intensity of earthquakes. The actual energy of the earthquake is more complex.

Ex. How does an earthquake of magnitude 8 compare to an earthquake of magnitude 4.5?

$8 = \log_{10} A_8$

$A_8 = 10^8$

$4.5 = \log_{10} A_{4.5}$

$A_{4.5} = 10^{4.5}$

$\frac{A_8}{A_{4.5}} = \frac{10^8}{10^{4.5}}$

$= 3162$

$\therefore$ the mag. 8 EQ. is 3162 times the intensity of the mag. 4.5 E.Q.

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Sound Loudness (decibel scale):

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where L is the loudness of the sound,
I is the sound intensity (energy), and
I_0 is the threshold of human hearing

note: The threshold, I_0, is not always necessary for a useful calculation.

Ex. How does the sound intensity of a rock concert compare to that of a subway (see p.498 for loudness values)?

$L_{RC} = 120 \qquad L_S = 90$

$120 = 10\log\left(\frac{I_{RC}}{I_0}\right) \qquad 90 = 10\log\left(\frac{I_S}{I_0}\right)$

$12 = \log_{10}\left(\frac{I_{RC}}{I_0}\right) \qquad 9 = \log\left(\frac{I_S}{I_0}\right)$

$10^{12} = \frac{I_{RC}}{I_0} \qquad 10^9 = \frac{I_S}{I_0}$

$I_{RC} = 10^{12} I_0 \qquad I_S = 10^9 I_0$

$\frac{I_{RC}}{I_S} = \frac{10^{12}\cancel{I_0}}{10^9\cancel{I_0}}$

$= 10^3$

$= 1000$

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Assigned Work:

p.499 # 1, 2, 3, 5a, 6a, 11, 13, 15
a
b

3. $L = 10\log\left(\frac{I}{I_0}\right)$ → threshold of hearing

Set $I = I_0$

$L = 10\log\left(\frac{I_0}{I_0}\right)$

$L = 10\log 1$

$L = 0 \rightarrow L = 10$
$\times 10$

$L = 10 \rightarrow L = 20$
$\times 10$

$L = 0 \rightarrow L = 60$, 6 steps
$10^6 \times$ intensity

$L = 10\log\left(\frac{10^6 I_0}{I_0}\right)$

$= 10\log_{10}(10^6)$

$= 10(6)$

$= 60$

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5 (a) $pH = 9$ $\quad pH = -\log H^+$

$9 = -\log H^+$

$-9 = \log_{10} H^+$

① equivalent expression

$10 \quad 10$

② both sides as exponents to base of log

$10^{-9} = H^+$

$3 = 5^x$

$\log_5 3 = \log_5 5^x$

$\log_5 3 = x$

$\frac{\log 3}{\log 5} = x$

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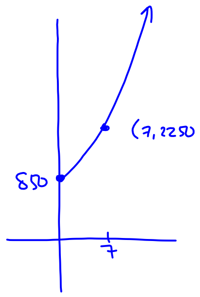
11 (a)

$y = ab^x + q$

$q = 0$ (asymptote)



$y = 850 b^x$

$b \doteq 1.15 \quad (7, 2250)$

Sub (12, 4500)

$4500 = 850 b^{12}$

$\frac{4500}{850} = b^{12}$

$\sqrt[12]{\frac{4500}{850}} = b$

$b \doteq 1.15$

$P(t) \doteq 850(1.15)^t$

(b) set $P(t) = 1700$

$1700 \doteq 850(1.15)^t$

$2 \doteq (1.15)^t$

$\log 2 \doteq \log(1.15)^t$

$t \doteq \frac{\log 2}{\log 1.15}$

$\doteq 4.96$

4.99 using $b = \sqrt[12]{\frac{4500}{850}}$

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13. 2.1% lost $\Rightarrow$ 97.9% remains

$$C(n) = 1(0.979)^n$$

$C(n)$: concentration; n: # of cycles

set $C(n) = 0.50$

$$0.50 = (0.979)^n$$

$$\log 0.50 = n \log 0.979$$

$$n = \frac{\log 0.50}{\log 0.979}$$

$$n \doteq 32.66$$

$\therefore$ it takes 33 cycles

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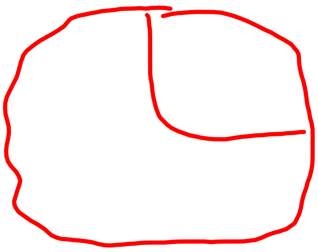
15. $A(t) = \underline{80}(10^{-0.023t})$

$$\underbrace{0.25(\cancel{80})}_{\text{75\% healed}} = \cancel{80}(10^{-0.023t})$$

75% healed

$\Rightarrow$ 25% still remains

80 cm^2



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