Products of Functions

Jan 7/2015

$$(f \times g)(x) = f(x) \times g(x)$$

"f times g of x"

To graph, pick an x-value and multiply the y-values from each of the original functions.

The domain of the combined function is the intersection of the domains of the original functions.

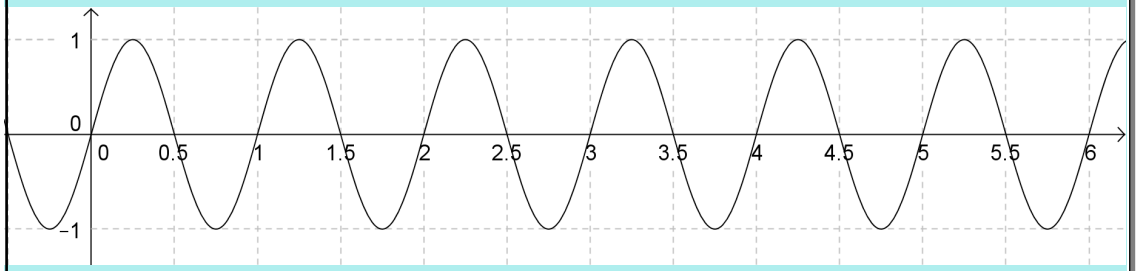
$$D_{f \times g} = D_f \cap D_g$$

When combining the functions algebraically, it is possible to have a situation where common factors will divide out of the numerator and denominator. Remember this will form a hole in a graph and a restriction in the domain.

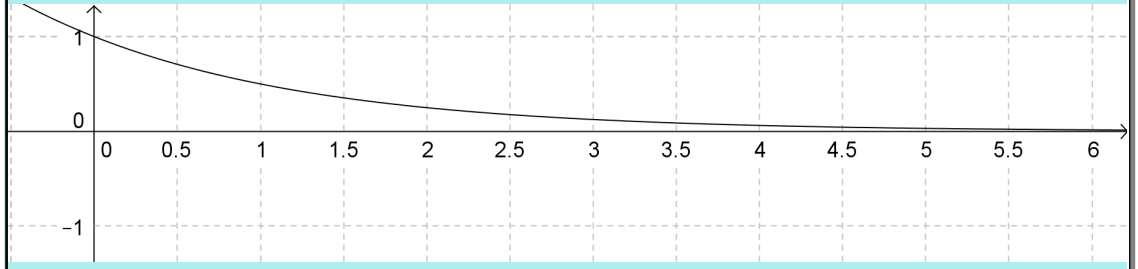
Jan 7-8:29 AM

Application - Damped Harmonic Motion

A perfect spring will oscillate forever:



More realistically, the oscillation will decay over time:



Jan 7-8:30 AM

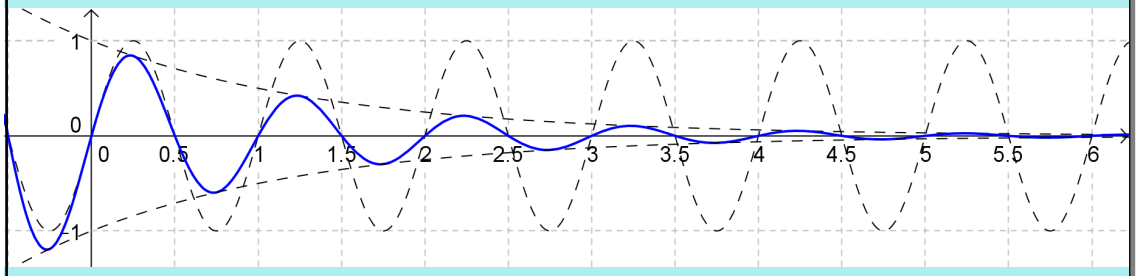
Application - Damped Harmonic Motion

Periodic Function: $f(x) = \sin(2\pi x)$

Exponential Decay: $g(x) = 2^{-x}$

Damped Harmonic Function:

$$(f \times g)(x) = 2^{-x} \sin(2\pi x)$$



Jan 7-8:30 AM

Ex.1 Given $f = \{(1,3),(2,-5),(3,7)\}$

$g = \{(2,-2),(3,3),(4,1)\}$

determine $f \times g$.

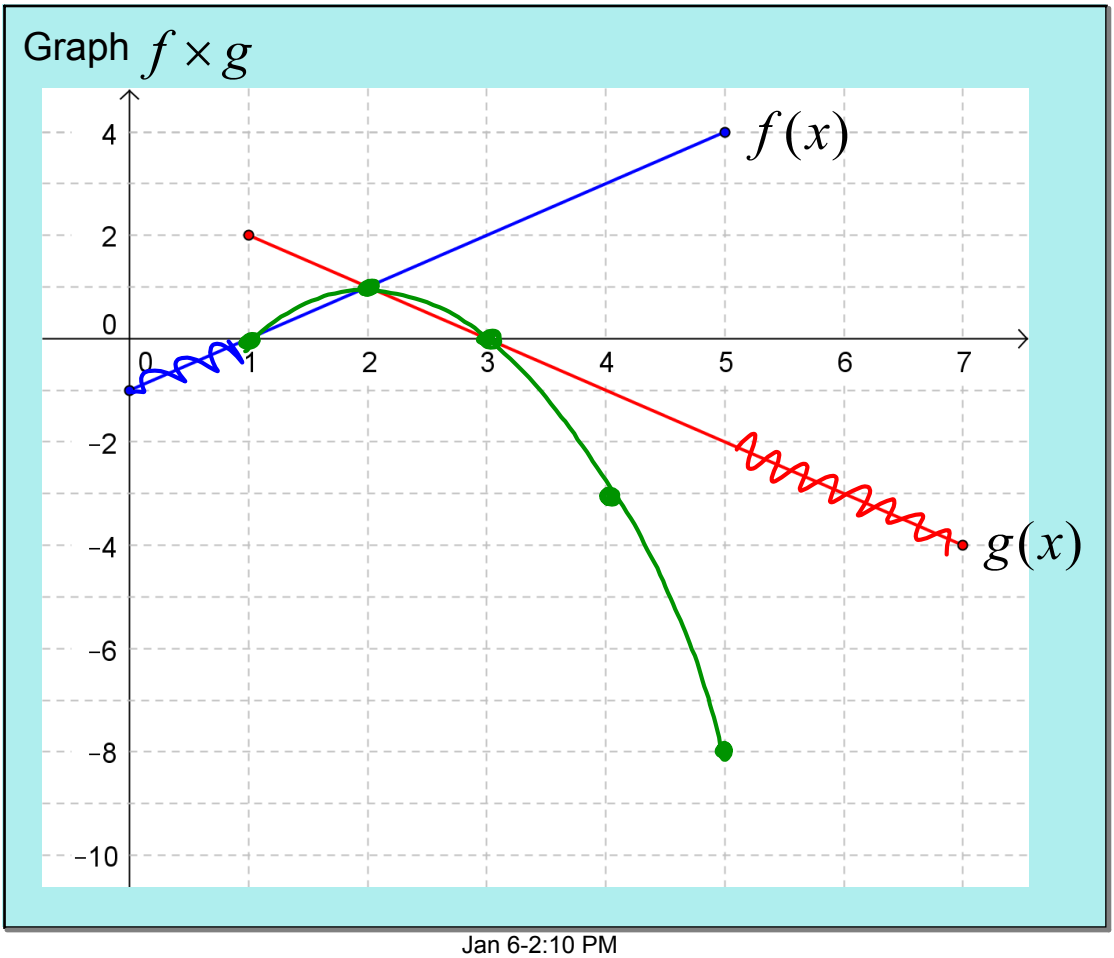
$D_f = \{1,2,3\}$ $D_g = \{2,3,4\}$

$D_{f \times g} = \{2,3\}$

$f \times g = \{(2,10),(3,21)\}$

Jan 6-8:57 AM

Graph $f \times g$



Jan 6-2:10 PM

Assigned Work:

p.537 # 1ae, 3, 4bef, 5bef, 6ef, 8ad, 11, 12

4(b) $f(x) = \sqrt{x+10}$ $g(x) = \sqrt{x+10}$

$(f \times g)(x) = (x+10)^{\frac{1}{2}}(x+10)^{\frac{1}{2}}$

$= (x+10)^1$

$= x+10$

5(b) $D_f = \{x \in \mathbb{R} \mid x \geq -10\} = D_g$

$D_{f \times g} = \{x \in \mathbb{R} \mid x \geq -10\}$

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8(d) $f(x) = \log(x^2+6x+9)$

$= \log(x+3)^2$

$D_f = \{x \in \mathbb{R} \mid x \neq -3\}$

$g(x) = \sqrt{x^2-1}$ $\quad x^2-1 \geq 0$

$x^2 \geq 1$

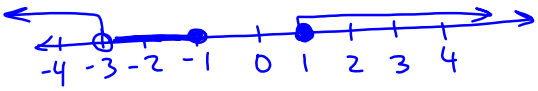
$|x| \geq \sqrt{1}$

$|x| \geq 1$

$x \geq 1, \; x \leq -1$

$D_g = \{x \in \mathbb{R} \mid x \leq -1, x \geq 1\}$

$D_{f \times g} = \{x \in \mathbb{R} \mid x \leq -1, x \geq 1, x \neq -3\}$



Jan 8-2:00 PM

11. $c(t) = 0.9^t$

$\ell(t) = 650 + 300t$

$f(t) = c(t)\,\ell(t)$

$= 0.9^t(650+300t)$

$f(t)$ a max?

$f(0) = 650$ $\quad f(9) = 1297$

$f(10) = 1273$ $\quad f(5) = 1269$

$f(100) = 0.8$ $\quad f(8) = 1312$

$f(20) = 808$ $\quad f(7) = 1315$

$f(6) = 1302$

$\therefore$ max occurs around 7 seconds.

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12. "If $f(x) \times g(x)$ is odd,
then $f(x)$ is odd and $g(x)$ is odd"

let $h(x) = f(x)\,g(x)$

assume $h(x)$ is odd

$h(x) = -h(-x)$

OR

$h(-x) = -h(x)$

$h(x) = -[f(-x)\,g(-x)]$

$= -f(-x)\,g(-x)$

$f(x)$ if odd

$= f(x)\,g(-x)$

$g(x)$ if even

$\therefore$ if $h(x)$ is odd then
f is odd and g is even
OR f is even and g is odd.

Jan 8-2:20 PM