Composition of Functions

Jan 9/2015

Suppose you were asked to graph $y = 2^{\sin x}$.

To make matters worse, suppose your calculator could only perform one operation at a time (i.e., you could perform the exponential operation, or the sine operation, but not both).

How would you get the points for your graph?

x	$\sin x$	2^x
0	$\sin(0) = 0$	$2^0 = 1$
1	$\sin(1) \doteq 0.84$	$2^{0.84} \doteq 1.79$
2	$\sin(2) \doteq 0.91$	$2^{0.91} \doteq 1.88$

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Composition of Functions

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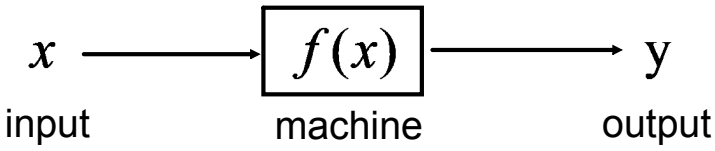
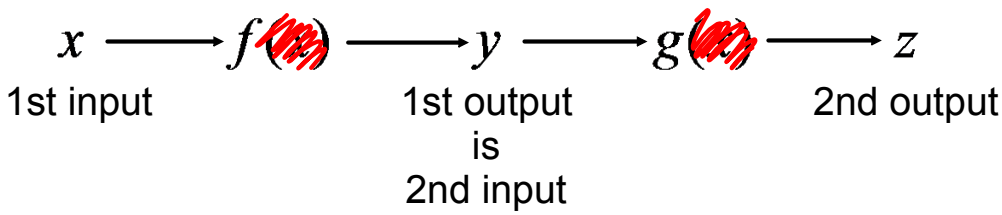
One way to view a function is as a machine, with an input (the independent variable, x) and an output (the dependent variable, y).



It is possible to connect multiple functions (machines) together, so the output of the first is the input to the second.



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A composition of functions occurs when the argument of a function is another function.

$$(f \circ g)(x) = f(g(x))$$

f composed with g

"f of g of x"

outer function (calculate 2nd)

inner function (calculate 1st)

Ex.1 Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$

(a) $(f \circ g)(x) = \sqrt{g(x)} = \sqrt{x^2 - 4}$

(b) $(g \circ f)(x) = (f(x))^2 - 4 = (\sqrt{x})^2 - 4 = x - 4$

$x \geq 0 \rightarrow x \geq 0$ carry forward restriction

Note: composition is not commutative

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Ex.1 Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$

(c) create a table of values for $(f \circ g)(x)$

(d) determine the domain of $f \circ g$

x	$g(x) = x^2 - 4$	x	$g(x)$	$f(x)$	$f(g(x)) = \sqrt{g(x)}$
-3	5	→	5		2.2
-2	0		0		0
-1	-3		-3		undef
0	-4		-4		"
1	-3		-3		"
2	0		0		0
3	5		5		2.2

for $(f \circ g)(x)$, restrict x

$D_{f \circ g} = \{x \in \mathbb{R} \mid x \leq -2, x \geq 2\}$

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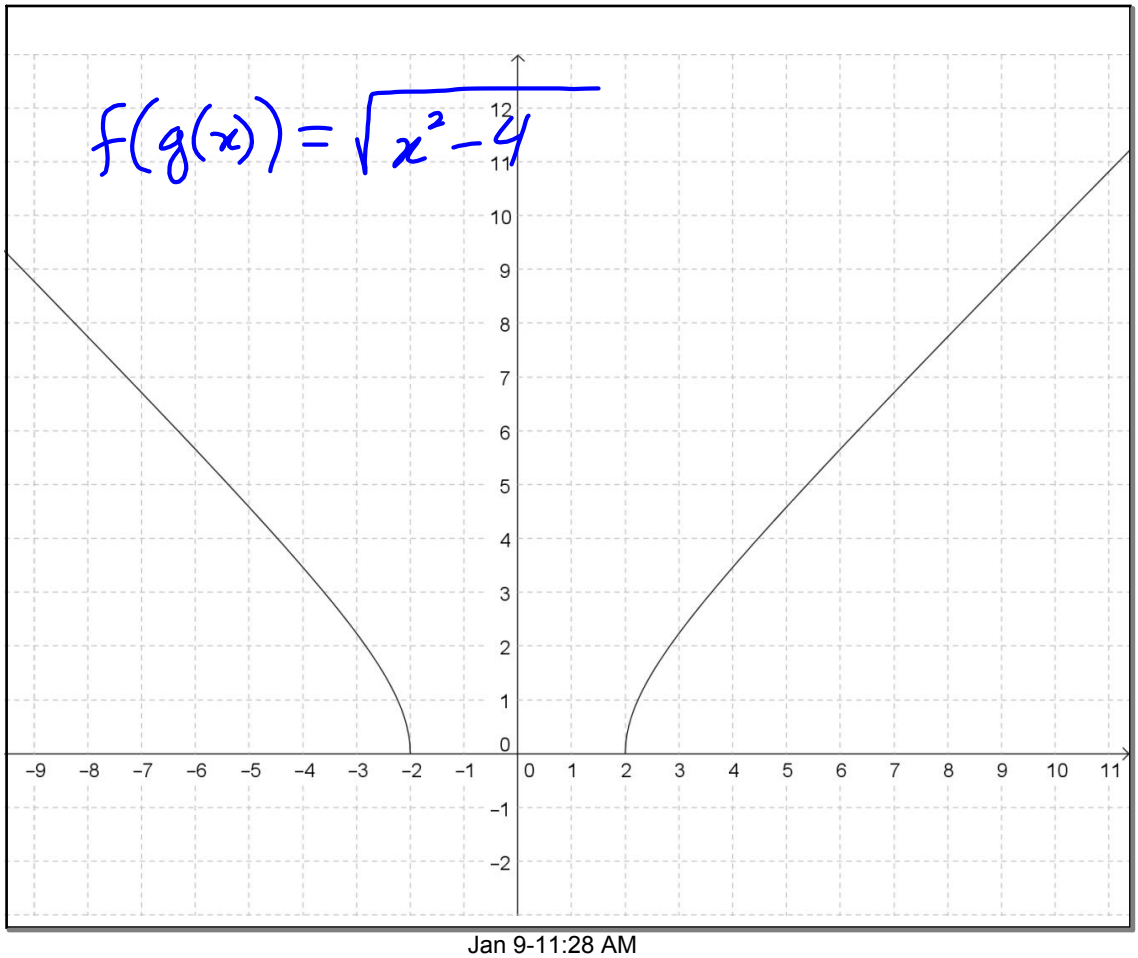

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When determining the domain of $f \circ g$:

(1) determine the range (output) of g

(2) determine the domain (input) of f

(3) restrict the domain of g so its range is within the domain of f

Assigned Work:

p.552 # 1, 2abf, 3, 5aef, 6def, 7cf, 10, 13

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Assigned Work:

p.552 # 1, 2abf, 3, 5aef, 6def, 7df, 10, 13

6(f) $f(x) = \sin x \quad g(x) = 5^{2x} + 1$

$f \circ g = f(g(x))$

$= \sin(g(x))$

$= \sin(5^{2x} + 1)$

$f(5) = \sin 5$

$f(g) = \sin g$

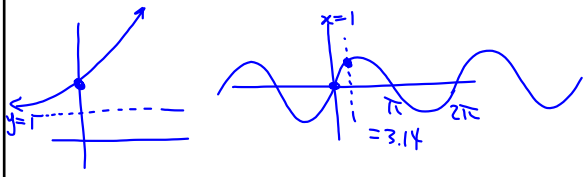
$f(g(x)) = \sin(g(x))$

$x \xrightarrow{5^{2x}+1} g(x) \xrightarrow{\sin x} f(x) \longrightarrow y$

D_1 $\quad R_1: y > 1$

$x \in \mathbb{R} \quad D_2: x > 1$

$R_2: -1 \le y \le 1$



$D_{f \circ g} = \{x \in \mathbb{R}\}$

$R_{f \circ g} = \{y \in \mathbb{R} \mid -1 \le y \le 1\}$

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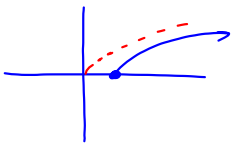
6 (d) $f(x) = 2^x \quad g(x) = \sqrt{x-1}$



$D_f = \{x \in \mathbb{R}\}$

$R_f = \{y \in \mathbb{R} \mid y > 0\}$

$D = \{x \in \mathbb{R} \mid x \ge 1\}$

$R = \{y \in \mathbb{R} \mid y \ge 0\}$

$f \circ g = f(g(x))$

$= 2^{g(x)}$

$= 2^{\sqrt{x-1}}$

$x \xrightarrow{\sqrt{x-1}} g(x) \xrightarrow{2^x} f(x) \longrightarrow y$

$x \ge 1 \longrightarrow y \ge 0$

$x \ge 0 \longrightarrow y \ge 1$

$D_{f \circ g} = \{x \in \mathbb{R} \mid x \ge 1\}$

$R_{f \circ g} = \{y \in \mathbb{R} \mid y \ge 1\}$

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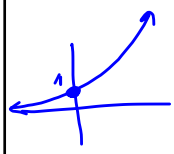
6 (d) $g \circ f = g[f(x)]$

$= \sqrt{f(x) - 1}$

$= \sqrt{2^x - 1}$



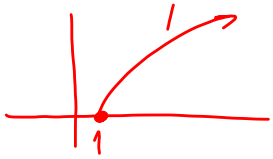
$x \xrightarrow{2^x} f(x) \xrightarrow{\sqrt{x-1}} g(x) \rightarrow y$

$x \in \mathbb{R} \rightarrow y > 0$

$x \geq 1$

$x \geq 0 \leftarrow y \geq 1$

$x \geq 1 \rightarrow y \geq 0$



$D_{g \circ f} = \{x \in \mathbb{R} \mid x \geq 0\}$

$R_{g \circ f} = \{y \in \mathbb{R} \mid y \geq 0\}$

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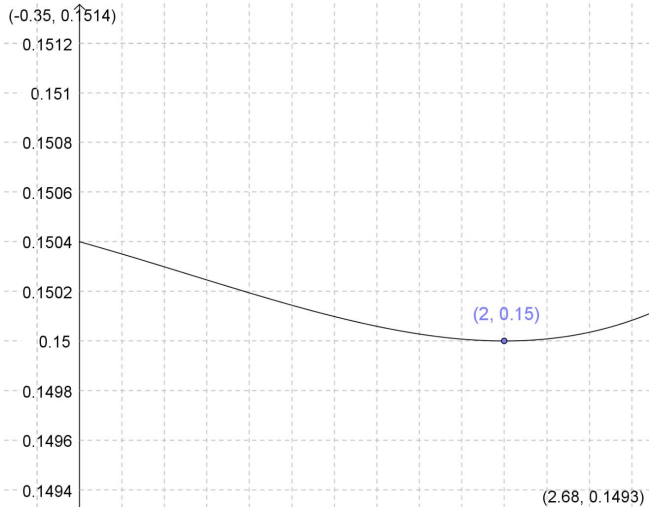
13. $v(t) = 40 + 3t + t^2$

$c(v) = \left(\frac{v}{500} - 0.1\right)^2 + 0.15$

$c(v) = \left(\frac{40 + 3t + t^2}{500} - 0.1\right)^2 + 0.15$



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