

EXAM REVIEW FOR MHF 4U
ADVANCED FUNCTIONS

Expressions

1. Factor fully.

a) $x^3 + 3x^2 - 25x - 75$

b) $x^3 + x^2 - 14x - 24$

c) $64x^3 + 27y^3$

d) $30x^3 + 17x^2 - 8x - 4$

e) $x^6 - 124x^3 - 125$

f) $x^7 - 8x^4 - 16x^3 + 128$

2. Simplify: $\sqrt{\frac{(x^a)^4}{x^{3a+b}} \cdot \frac{(x^b)^4}{x^{a+3b}}}$

3. Find the exact value of the following.

a) $7^{\log_7 \sqrt{5}}$

b) $\log_{64} \sqrt[6]{8}$

c) $\log_8 6 - \log_8 3 + \log_8 4$

d) $\log_9 (3^7 \cdot \sqrt[5]{81})$

4. Write as a single logarithm: $a \log_5 (x-7) - \frac{2}{3} \log_5 w + 2$

5. Let $f(x) = \{(3,2), (5,1), (7,4), (9,3), (11,5)\}$ and $g(x) = \{(1,3), (2,5), (3,7), (4,9), (5,11)\}$.

Determine:

a) $f(g(3))$

b) $(g \circ f)(9)$

c) $(f-g)(x)$

d) $(f+g)(x)$

6. Convert to degrees.

a) $\frac{11\pi}{15}$ radians

b) 56 radians

7. Convert to radians.

a) 420°

b) -24°

8. Find the exact value of the following.

a) $\cos \frac{3\pi}{4}$

b) $\csc \left(\frac{-3\pi}{2} \right)$

c) $\tan \frac{11\pi}{6}$

d) $\sin \frac{7\pi}{12}$

e) $\sec \left(\frac{5\pi}{6} \right) \cos \left(\frac{7\pi}{4} \right) - \cot \left(\frac{-\pi}{3} \right)$

9. Given: $\sin A = \frac{-6}{7}$, $\frac{3\pi}{2} \leq A \leq 2\pi$, and $\tan B = \frac{2}{3}$, $\pi \leq B \leq \frac{3\pi}{2}$

Find the exact value of the following.

a) $\sec A$

c) $\sin(A+B)$

b) $\cos 2B$

d) $\tan(A-B)$

10. Given: $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 + 8$

Find:

a) $(f-g)(x)$

e) $(g \circ (g))(x)$

i) $(g-f)(3)$

b) $\left(\frac{g}{f} \right)(x)$

f) $f^{-1}(x)$

j) $(fg)(-1)$

c) $(f \circ g)(x)$

g) $g^{-1}(x)$

k) $(f \circ g)(5)$

d) $(g \circ f)(x)$

h) $(f \circ f^{-1})(x)$

l) $(g \circ f)(5)$

Equations, Inequalities & Identities

11. Solve. Exact answers are required, where possible. Otherwise, express answers correct to one decimal place. Where necessary, state restrictions.

- a) $x^3 - 3x^2 = 4x - 12$
 b) $x^3 - 5x = 5x^2 - 1$
 c) $x^3 + 4x^2 + 9x + 10 = 0$
 d) $x + \frac{1}{x-4} = 0$
 e) $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2 - 4x + 3}$
 f) $4(7^{x-2}) = 8$
 g) $\log_4(x+3) = 2$
 h) $\log_7(x+2) = 1 - \log_7(x-4)$
 i) $\log_9(x-5) + \log_9(x+3) = 1$
 j) $\log_5(x+1) + \log_5 2 - \log_5(x+3) = \log_5(x-1)$
 k) $5 \cdot 8^{x+2} = 5^{7x}$
 l) $(4^2)(2^{2x-3}) = (16^{x-2})\left(\frac{1}{\sqrt{2}}\right)$
 m) $3^{2x} - 2(3^x) - 15 = 0$
 n) $\sin^2 x - 2\sin x - 3 = 0$ ($0 \leq x \leq 2\pi$)
 o) $\cos 2x = \cos x$ ($0 \leq x \leq 2\pi$)
 p) $\sqrt{2} \tan x \cos x = \tan x$ ($0 \leq x \leq 2\pi$)
 q) $2\cos 2x = 1$ ($0 \leq x \leq 2\pi$)

12. Solve.

- a) $x(x+1)(x-2)(x-4) > 0$
 b) $(x+7)^2(x-3)^3 < 0$
 c) $2x^3 + 3x^2 - 11x \geq 6$
 d) $\frac{x+2}{x^2-9} \geq 0$
 e) $\frac{5}{x+3} + \frac{3}{x-1} < 0$

13. Prove.

- a) $\cos \theta + \sin \theta = \frac{1 + \tan \theta}{\sec \theta}$
 b) $\frac{1}{1 - \sec \theta} + \frac{1}{1 + \sec \theta} = -2 \cot^2 \theta$
 c) $\cos^2 2\theta - \cos^2 \theta = \sin^2 \theta - \sin^2 2\theta$
 d) $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) = -\cot x$
 e) $\frac{\sin(\pi - x) \cos(\pi + x) \tan(2\pi - x)}{\sec\left(\frac{\pi}{2} + x\right) \csc\left(\frac{3\pi}{2} - x\right) \cot\left(\frac{3\pi}{2} + x\right)} = \sin^4 x - \sin^2 x$
 f) $\cos(x+y) \cos(x-y) = \cos^2 x + \cos^2 y - 1$

14. If $\log_b a = \frac{1}{x}$ and $\log_a \sqrt{b} = 3x^2$, show that $x = \frac{1}{6}$.

15. If $h^2 + k^2 = 23hk$, where $h > 0$, $k > 0$, show that $\log\left(\frac{h+k}{5}\right) = \frac{1}{2}(\log h + \log k)$

Graphs

16. Determine whether each of the following functions are even, odd or neither.

- a) $f(x) = \frac{1}{x^3 + 1}$ b) $g(h) = 2x^4 + 3x^2$ c) $h(x) = \left(\frac{1}{x^3 + x}\right)^5$

17. Graph the following functions. Determine and label all key features.

- a) $y = -x(x-3)(x+4)$
 b) $y = (x-2)^2(x+3)^3$
 c) $y = \frac{-2}{x-1}$
 d) $y = \frac{5x-3}{2x+1}$
 e) $y = 3^{x+2} - 1$
 f) $y = \log_2(8x^2)$
 g) $y = 2 \sin\left(x - \frac{\pi}{3}\right)$, ($-2\pi \leq x \leq 2\pi$)
 h) $y = \cos\left(\frac{1}{2}x + \frac{\pi}{4}\right) - 1$, ($-2\pi \leq x \leq 2\pi$)

18. When is the function, $f(x) = \frac{4}{x-1} - 3 + \frac{-3x^2}{5-4x-x^2}$, below the horizontal asymptote?

19. State the range, period, amplitude, phase shift and equations of the asymptotes for each of the following functions for $0 \leq x \leq 2\pi$. (State only the properties that each function has.)

a) $y = -3\cos\left(3x - \frac{\pi}{4}\right) - 2$ b) $y = \cot\left(x - \frac{\pi}{6}\right)$

Applications

20. When $x^4 - 4x^3 + ax^2 + bx + 1$ is divided by $(x-1)$, the remainder is 7. When it is divided by $(x+1)$, the remainder is 3. Determine the values of a and b .

21. An open box, no more than 5 cm in height, is to be formed by cutting four identical squares from the corners of a sheet of metal 25 cm by 32 cm, and folding up the metal to form sides. The capacity of the box must be 1575 cm^3 . What is the side length of the squares removed?

22. Consider all rectangles with an area of 200 m^2 . Let x be the length of one side of the rectangle.

- Express the perimeter as a function of x .
- Find the dimensions of a rectangle whose perimeter is 70 m.

23. Determine the intercepts, holes and the equations of all asymptotes with behaviour of

$y = \frac{x^3 - 2x^2 - x + 2}{x^2 - x - 6}$ then sketch.

24. Estimate instantaneous rate of change of each function at the given x value using a centered interval of ± 0.001 .

a) $f(x) = x^3 + x^2$ at $x = 2$ b) $f(x) = -x^4 + 1$ at $x = 3$

25. The population of a town is modelled by $P(t) = 6t^2 + 110t + 3000$, where P is the population and t is the number of years since 1990. Find the average rate of change in population between 1995 and 2005.

26. Energy is needed to transport a substance from outside a living cell to inside the cell. This energy is measured in kilocalories per gram molecule, and is given by: $E = 1.4(\log C_1 - \log C_2)$, where C_1 represents the concentration of the substance outside the cell and C_2 represents the concentration of the substance inside the cell.

- Rewrite the formula as a single logarithm.
- Find the energy needed to transport the exterior substance into the cell if the concentration of the substance outside the cell is double the concentration inside the cell.
- What is the sign of E if $C_1 < C_2$? Explain what this means in terms of the cell.

27. A ferris wheel with a radius 10 m makes 2 rotations in 4 minutes. What is the speed of the ferris wheel in meters per second.

28. A circular arc has length 3 cm, and the radius of the circle is 2 cm. What is the measure of the angle subtended by the arc, in both radians and in degrees?

Answers to MHF4U Exam Review

Check

1.a. $(x-5)(x+5)(x+3)$

b. $(x-4)(x+3)(x+2)$

c. $(4x+3y)(16x^2-12xy+9y^2)$

d. $(5x+2)(2x-1)(3x+2)$

e. $(x+1)(x-5)(x^2-x+1)(x^2+5x+25)$

f. $(x+2)(x-2)^2(x^2+4)(x^2+2x+4)$

2. $x^{\frac{-a}{2} + \frac{b}{2}}$

3a. $\sqrt{5}$ b. $\frac{1}{12}$ c. 1 d. $\frac{39}{10}$

4. $\log_5\left(\frac{25(x-7)^a}{w^{\frac{2}{3}}}\right)$

5a. 4 b. 7 c. $\{(3,-5), (5,-10)\}$

d. $\{(3,9), (5,12)\}$

6a. 132° b. $\frac{10080}{\pi} \doteq 3208.6^\circ$

7a. $\frac{7\pi}{3}$ b. $\frac{-2\pi}{15}$

8a. $\frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$ b. 1

c. $\frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$ d. $\frac{\sqrt{6} + \sqrt{2}}{4}$

e. $\frac{\sqrt{3} - \sqrt{6}}{3}$

9a. $\frac{7\sqrt{13}}{13}$ b. $\frac{5}{13}$ c. $\frac{18\sqrt{13} - 26}{91}$

d. $\frac{-2(9 + \sqrt{3})}{3(4 + \sqrt{3})} = \frac{10\sqrt{3} - 66}{39}$

10a. $\frac{41 - 8x + 5x^2 - x^3}{x-5}$

b. $x^3 - 5x^2 + 8x - 40$

c. $\frac{1}{x^2 + 3}$

d. $\frac{8x^2 - 80x + 201}{(x-5)^2}$

e. $x^4 + 16x^2 + 72$ f. $\frac{1}{x} + 5$

g. $\pm\sqrt{x-8}$ h. x i. $\frac{35}{2}$ j. $\frac{-3}{2}$

k. $\frac{1}{28}$ l. undefined

11a. 3, 2, -2 b. -1, $3 \pm 2\sqrt{2}$

c. -2, $-1 \pm 2i$ d. $2 \pm \sqrt{3}$ e. $\frac{-1}{2}$

f. $\frac{\log 2}{\log 7} + 2 \doteq 2.36$ g. 13

h. 5 i. 6 j. no solution

k. $\frac{2\log 8 + \log 5}{7\log 5 - \log 8} \doteq 0.3$ l. $\frac{19}{4}$

m. $\frac{\log 5}{\log 3} \doteq 1.5$ n. $\frac{3\pi}{2}$

o. $0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$ p. $0, \frac{\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$

q. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12a. $x \in (-\infty, -1) \cup (0, 2) \cup (4, +\infty)$

b. $x \in (-\infty, -7) \cup (-7, 3)$

c. $x \in \left[-3, \frac{-1}{2}\right] \cup [2, +\infty)$

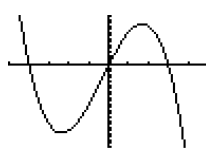
d. $x \in [-3, -2] \cup [3, +\infty)$

e. $x \in (-\infty, -3) \cup \left(-\frac{1}{2}, 1\right)$

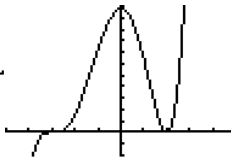
13. - 15. (proofs vary)

16a. neither b. even c. odd

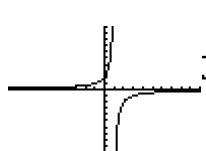
17a.



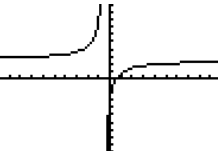
b.



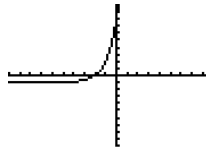
c.



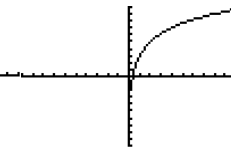
d.



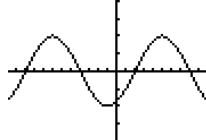
e.



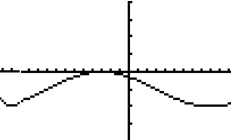
f.



g.



h.



18. $x \in (-5, 1) \cup \left(\frac{35}{8}, +\infty\right)$

19a. $R: \{y | -5 \leq y \leq 1, y \in \mathbb{R}\}$,

Period = $\frac{2\pi}{3}$, amplitude = 3,

Phase shift = $\frac{\pi}{12}$ right, No

Asymptotes

b. $R: \{y \in \mathbb{R}\}$, Period = π ,

no amplitude, phase shift = $\frac{\pi}{6}$

right,

Asymptotes: $x = \frac{\pi}{6}$, $x = \frac{7\pi}{6}$

20. a=3, b=6

21. 3.5 cm

22a. $P(x) = 2x + \frac{400}{x}$

b. 27.8 x 7.2 m

23. x-int=1, -1, 2

y-int = $\frac{-1}{3}$

VA: x=3, x=-2

Obl A: y=x-1

24a. 16 b. -108

25. 230 people/year

26a. $E = 1.4 \log\left(\frac{C_1}{C_2}\right)$

b. 0.42 c. neg - cell gains energy

27. $\frac{\pi}{12}$ m/s

28. $\theta = 1.5 \text{ rad}$ or $\theta = \frac{270}{\pi}$ degrees