MHF4U – Final Exam Review

Polynomial Functions

1. Determine the zeros of the function $f(x) = 3(x^2 - 25)(4x^2 + 4x + 1)$.

2. The remainder when $x^4 + Cx^2$ is divided by $x + 2$ is 28.

3. Determine the equation of:

   a. The cubic function with zeros 1, -2, -5 and y-intercept 10.
   b. The quartic function with zeros 2, -1, 3 (order 2) and passing through (4,-10)

4. By calculating the Finite Differences, determine
   a. Which type of polynomial function best models this relationship.
   b. Determine the minimum and maximum number of zeros and turning points for this type of function.

x	-2	-1	0	1	2	3	4
$f(x)$	1	0	1	16	81	256	625

5. a) Without using division (long or synthetic), find the remainder when $2x^3 - 4x + 7$ is divided by $2x - 1$
   b) $x - 5$ is a factor of $4x^3 - 5x + kx - 2$. Find k.
   c) Divide using long division $(3y^3 - 2y^2 + 12y - 9) \div (y^2 + 2)$

6. Factor fully
   a) $64x^3 + 125y^3$ b) $3x^3 + 8x^2 + 3x - 2$ c) $x^4 - 2x^3 + 2x - 1$
   d) $8x^3 - 4x^2 - 2x + 1$ e) $2x^3 + 13x^2 + 23x + 12$

7. Solve for $x, x \in C$
   a) $x^3 - 7x + 6 = 0$ b) $2x^3 + x^2 - 8x - 4 = 0$ c) $9z^3 - 4z = 0$
   e) $x^3 - 4x^2 + 2x + 3 = 0$ f) $3x^3 - 7x^2 = 2 - 6x$ g) $x^2(2x - 1) = 2x - 1$

8. Solve for t or x
   a) $t^3 - 2t^2 - 3t > 0$ b) $t^3 + 2t^2 - 3t < 0$ c) $x(x - 2)(x + 1)(x + 5) \geq 0$

9. Find the equation of a polynomial function in standard form, with the given the roots:
   a) 3, -2, $\frac{1}{2}$ b) 1, 1, 6 c) 3, $1 \pm i$ d) 0, $2 \pm \sqrt{3}$, $\frac{-2}{3}$

10. For the function $g(x) = -2x^3(x - 2)^2$
    a. State the degree of the function and comment on its end behaviour.
    b. State the zeros and with the aid of a table of values, sketch the function.
    c. State from the graph, the interval(s) where $g(x)$ is decreasing.
    d. Find, algebraically where $g(x) < 0$. Illustrate, in colour, on the graph.
    e. State the coordinates of any local/absolute max/min points.

Rational Functions

1. Find the vertical and horizontal asymptotes for $f(x) = \dfrac{x+2}{3x-2}$

2. Given: $f(x) = \dfrac{x^2}{x^3 - 2x^2 - 5x + 6}$. Find the domain, intercepts and vertical and horizontal asymptotes. Sketch the graph.

3. Given: $g(x) = \dfrac{x-2}{x^2 + 5x + 6}$

   a. Determine x and y intercepts.
   b. State the domain.
   c. State any asymptotes.
   d. Are there any holes?

4. Given: $\dfrac{x-3}{x^2 - 9}$
   a. Find the x and y intercepts.
   b. Find the domain.
   c. Find any asymptotes.
   d. Are there any holes?

5. Create a function that has a graph with the given features.

   a. Vertical Asymptote at $x = -2$ and horizontal asymptote at $y = 1$
   b. Two vertical asymptotes at $x = 3$ and $x = -4$, with a hole at $x = 1$
   c. Vertical Asymptote at $x = -3$, horizontal asymptote at $y = 0$, no x-intercept and a y intercept at –2
   d. No vertical asymptote and x-intercepts at 2 and –1

6. Graph $f(x) = \dfrac{x^2 - 16}{x}$. Find the domain, the intercepts, the location of any asymptotes and describe the function's behaviour near these asymptotes.

7. Simplify and state restrictions.

   a. $\dfrac{x+1}{x} - \dfrac{x}{x+1}$ b. $\dfrac{x - x^3}{x^2 - 2x - 3} \div \dfrac{x^2 - x}{2x - 6}$

8. Solve and state any restrictions.

   a. $\dfrac{x-1}{x-3} = \dfrac{x+3}{x+4}$ b. $\dfrac{x}{x-1} + \dfrac{1}{x+1} = \dfrac{2}{x^2 - 1}$ c. $0 = 8\pi r - \dfrac{2000}{r^2}$

   d. $0 = \dfrac{5}{\sqrt{x^2 + 4}} - 2$ e. $0 = 2x(x^2 + 1)^{-\frac{1}{2}} - 1$ f. $0 = 2 - \dfrac{10}{x^2}$

9. The estimate revenue and cost functions for the manufacture of a new product are R(x)=-2x^2+15x and C(x)=5x+8.
   a) Express the average profit function $AP(x) = \frac{P(x)}{x}$, in two different forms.
   b) Explain what can be determined from each form.
   c) What is the domain of the function in this context?
   d) What are the break-even quantities?

10. The value of a car, t years after it is bought, is modeled by $V(t) = \frac{2100 + 8t}{1 + 0.5t} + 150$.
   a) State the domain of V(t), determine any intercepts as asymptotes, and sketch the function.
   b) What will the car be worth in the long run?
   c) After how many years will the car be worth $1500?
   d) Find the average rate of change in the value of the car between 2 and 5 years and the instantaneous rate of change at 2 years.

11. Sketch $f(x) = -x^2 + 9$ and its reciprocal on the same grid.

Trigonometry Review

1. Sketch $-2\pi \le \theta \le 2\pi$ of:

   a) y = 2 sin3$\left(\theta - \frac{\pi}{3}\right)$-1
   b) y = -4 cos$\left(\frac{1}{2}\theta - \pi\right) + 3$
   c) y = $\frac{1}{2}$sin2$\left(\theta + \frac{\pi}{4}\right)$
   d) y = -3 cos$(4\theta + \pi)$-2

2. Sketch f(θ) = 2 sin $\left(\theta + \frac{\pi}{6}\right)$ on the interval $\left[\frac{-\pi}{6}, \frac{5\pi}{6}\right]$.
   a) Find the average rate of change (to 5 decimals) of the function from $\frac{\pi}{3} to \frac{2\pi}{3}$. What does this tell us about the graph?
   b) Find the instantaneous rate of change (to 5 decimals) of the function at $\frac{2\pi}{3}$. What does this tell us about the graph?

3. State two co-terminal angles for each of the following. Your answers must include π.

   a) $\frac{2\pi}{3}$
   b) $\frac{11\pi}{6}$
   c) $\frac{\pi}{5}$
   d) $-\frac{3\pi}{4}$

4. Find each function value.

   a) cscθ if sin$\theta = \frac{3}{\sqrt{5}}$
   b) cot θ if csc$\theta = \frac{7}{3}$

5. Find each of the following. Use exact values only.

a) $\sin\frac{7\pi}{4}$ b) $\sec\frac{2\pi}{3}$ c) $\cot\frac{11\pi}{6}$ d) $\cos\frac{3\pi}{4}$

6. Find all values for $0 \le \theta \le 2\pi$.

a) $\cos\theta = \frac{\sqrt{3}}{2}$ b) $\sec\theta = -\sqrt{2}$ c) $\cot\theta = -1$ d) $\tan\theta = \sqrt{3}$

7. Solve for $0 \le \theta \le 2\pi$.

a) $\cos 2\theta = \frac{\sqrt{3}}{2}$ b) $\sin 2\theta = \frac{1}{2}$ c) $\tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}}$

d) $\cos\left(\frac{\theta}{2}\right) = -\frac{1}{2}$ e) $\sin\left(\theta - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$ f) $\cos\left(\theta + \frac{\pi}{3}\right) = \frac{1}{2}$

8. Solve for $0 \le x \le 2\pi$.

a) $\cos^2 x = \frac{3}{4}$ b) $2\sin^2 x + \sin x - 1 = 0$ c) $10\cos^2(2x) + 7\cos(2x) = 6$

d) $4\cos^2(2x) - 1 = 0$ e) $3\tan^2 x = 1$ f) $2\tan^2 x + \tan x - 3 = 0$

g) $6\cos^2 x - \sin x - 4 = 0$ h) $2\cos x = 1 - \sin^2 x$ i) $2\sin^2 x = -\cos x + 1$

9. Write as a trig function of the related angle and evaluate:

a) $\cos\left(\frac{5\pi}{6}\right)$ b) $\tan\left(2\pi - \frac{\pi}{4}\right)$

10. Write as a trig function of the co-related (co-function) angle and evaluate.

a) $\sec\left(\frac{11\pi}{3}\right)$ b) $\sin\left(\frac{3\pi}{2} - \frac{\pi}{4}\right)$

11. Write as a single trig function.

a) $\sin A\cos B - \cos A\sin B$ b) $\cos 2A\cos A - \sin 2A\sin A$ c) $\frac{2\tan x}{1-\tan^2 x}$

d) $\cos^2 3M - \sin^2 3M$ e) $10\sin x\cos x$ f) $1 - 2\sin^2\left(\frac{2\theta}{3}\right)$

g) $-4\sin\frac{x}{2}\cos\frac{x}{2}$ h) $2\cos^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right)-1$ i) $2\tan a\cos^2 a$

j) $\frac{\tan 3x-\tan 4x}{1-\tan 3x\tan 4x}$

12. Evaluate the following.

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}-\cos\frac{\pi}{4}\sin\frac{\pi}{12}$ b) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

c) $\frac{\tan\frac{\pi}{12}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{12}\tan\frac{\pi}{6}}$ d) $\cos^2\frac{\pi}{12}+\sin^2\frac{\pi}{12}$

e) $\cos^2\frac{\pi}{12}-\sin^2\frac{\pi}{12}$ f) $\left(\cos\frac{\pi}{12}+\sin\frac{\pi}{12}\right)^2$

g) $\sin\frac{\pi}{12}$ h) $4\cos^2\left(\frac{\pi}{8}\right)-2$

i) $\tan\frac{11\pi}{12}$ j) $\sin\left(\frac{3\pi}{2}-\frac{\pi}{3}\right)$

13. Simplify.

a) $\frac{(\sin x+\cos x)(\sin x-\cos x)+1}{\sin^2 x}$ b) $\frac{\tan^2 x\sin^2 x}{\sec^2 x}$

c) $\frac{\sin(x-30°)+\cos(60°-x)}{\sin x}$ d) $\frac{\sin 2a}{\cos a}$

e) $2\sin^2 a+\cos 2a$ f) $(\sin x+\cos x)^2-\sin 2x$

g) $2\sin x\cos^3 x-2\sin^3 x\cos x$

14. Develop a formula for $\sin 3\theta$ in terms of $\sin\theta$.

15. If $\sin x=\frac{4}{5}$ and $\cos y=-\frac{12}{13}$, where $0\le x\le\frac{\pi}{2}$ and $\frac{\pi}{2}\le y\le\pi$, find the following.

a) sin(x – y) b) tan(x + y) c) sec(x – y)

16. If $\cot x=-\frac{2}{3}$ for $\frac{\pi}{2}<x<\pi$, find the following.

a) tan 2x b) cos 2x c) csc 2x d) sin 4x e) tan $\frac{x}{2}$

17. Prove the following identities.

a) $\dfrac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$

b) $\dfrac{\cos^4 x - \sin^4 x}{1-\tan^4 x} = \cos^4 x$

c) $\dfrac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = 1 + \sin x \cos x$

d) $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$

e) $\dfrac{\cos(x-y)}{\cos x \cos y} = 1 + \tan x \tan y$

f) $\dfrac{\csc\left(\dfrac{\pi}{2}+x\right)}{\csc(2\pi - x)} + \dfrac{\sin(\pi - x)}{\sin\left(\dfrac{3\pi}{2} - x\right)} = 2\cot\left(\dfrac{3\pi}{2}+x\right)$

g) $\dfrac{1-\cos 2x}{\tan x} = \sin 2x$

h) $2\cos x - 2\cos^3 x = \sin x \sin 2x$

i) $\sin\left(\dfrac{\pi}{4}+x\right) + \sin\left(\dfrac{\pi}{4}-x\right) = \sqrt{2}\cos x$

j) $\cot\theta + \tan\theta = 2\csc 2\theta$

k) $\sec t = \dfrac{\sin 2t}{\sin t} - \dfrac{\cos 2t}{\cos t}$

18. Sketch y = cot x for $-2\pi \le x \le 2\pi$.

19. a) State the equations of the vertical asymptotes for
   i) y = sec x for $x \varepsilon \Re$
   ii) y = tan x for $x \varepsilon \Re$

   b) State the local minimums for y = csc x for $x \varepsilon \Re$.
   c) State the period of y = tan 2x.

Exponential and Logarithmic Functions

1. Sketch the following
   (a) $y = 3(2)^x$ (b) $y = -4(2)^x$ (c) $y = -(2)^{3x}$ (d) $y = 2^{2x+6}$
   (e) $y = 2^{-x} + 4$ (f) $y = 2^{2x-4} - 3$ (g) $y = 4\left(\dfrac{1}{2}\right)^x$

2. Sketch the following
   (a) $2\log x$ (b) $y = -\log x + 2$ (c) $y = \log(3x)$
   (d) $y = \log\ (x+4)$ (e) $y = \log(2x-6)$ (f) $y = \log\left(\dfrac{1}{3}x + 2\right)$ g) $y = -\log(-2x) + 2$

3. Express in logarithmic form

   (a) $5^2 = 25$ (b) $6^0 = 1$ (c) $49^{\frac{1}{2}} = 7$ (d) $125^{\frac{2}{3}} = 25$

4. Express in exponential form

   (a) $\log_3 1 = 0$ (b) $\log_6\left(\frac{1}{36}\right) = -2$ (c) $\log_9 27 = \frac{3}{2}$

5. Evaluate the following

   (a) $\log_7 49$ (b) $\log_2 16$ (c) $\log_3 \sqrt{27}$ (d) $\log_2 \sqrt[3]{32}$ (e) $\log_{17} 1$ (f) $\log_3 \frac{1}{81}$

   (g) $6^{-2\log_6 8}$ (h) $7^{2\log_7 6}$ (i) $\log_3\left(9\sqrt{27}\right)$ (j) $\log_4(64\times\sqrt[3]{16})$ (k) $\log_5\left(\frac{\sqrt[3]{25}}{625}\right)$

   (l) $\log_{12} 576 - \log_{12} 4$ (m) $\log_6 18 + \log_6 12$

6. Solve

   (a) $\log x = \log 5 + 2\log 3$ (b) $\log\sqrt{x} = \log 1 - 3\log 2$ (c) $\log_3 x - \log_3 4 = \log_3 12$

   (d) $\log_6(x+3) + \log_6(x-2) = 1$ (e) $\log_7 x = 3\log_7 4$ (f) $\log_5(7x+1) - \log_5(x-1) = 2$

7. Express as a single log:

   (a) $\frac{1}{2}\log_5 x + \frac{1}{3}\log_5 y - \frac{1}{4}\log_5 z$ (b) $\frac{1}{2}[\log_4 x + 3\log_4 y] - 2[\log_4 a + \log_4 b]$

8. Solve

   (a) $3^{2x-1} = 5$ (b) $2^{2x} + 3(2^x) - 10 = 0$ (c) $6^{2x} - 2(6^x) - 15 = 0$ (d) $\left(\frac{1}{64}\right)^{x+2} = 8^{2x}$

   (e) $6(10^{2x}) + 10^x - 2 = 0$

9. A sample of 500 cells in a medical research lab doubles every 20 min.
   a) Determine a formula for the number of cells at time t, where t is measured in minutes.
   b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places.

10. In 1987 there were about 130 000 cell phone users in Canada. In 1999, there were about 10 million cell phone users. What is the percent increase per year? Answer correct to 2 decimal places.

11. A sample of 700 cells in a medical research lab triples every 30 min.
    a) Determine a formula for the number of cells at time t.
    b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places

12. A sample of radioactive iodine-131 atoms has a half-life of about 8 days. Suppose that 1 000 000 iodine-131 atoms are initially present.
    a) Determine a formula for the number of atoms at time t, where t represents number of days
    b) How long will it take for the sample to reach 180 000 atoms? Answer correct to 2 decimal places.

13. A new car costs $23 000. In 5 years it will be worth $9500. What is the rate of depreciation per year? Answer in percent, correct to 2 decimal places.

14. Most of Canada's earthquakes occur along the west coast. In 1949, there was an earthquake in the Queen Charlotte Islands that had a magnitude of 8.1 on the Richter Scale. In 1997 there was an earthquake in south-western B.C. with a magnitude of 4.6 on the Richter Scale. How many times as intense as the 1997 earthquake was the 1949 earthquake? Answer correct to 2 decimal places.

15. The loudness L in decibels ($\frac{1}{10}$ of a bel) of a sound of intensity *I* is defined to be

$$L = 10\log\frac{I}{I_0}$$

    where I_0 is the minimum intensity audible to the human ear

    a) The loudness level of a heavy snore is 69 dB. How many times is this more intense than conversational speech at 60 dB? Answer correct to 2 decimal places.
    b) Sound is 316 times less intense if earplugs are worn. What would the decibel level of snoring be if earplugs were worn? Answer correct to the nearest dB.

16. Given the function $y = 4^x$, determine:
    a. the average rate of change from t=5 seconds to t=6 seconds.
    b. the instantaneous rate of change at t=5 seconds.